International Journal of Frontiers in
Engineering and Technology Research
Journal homepage: https://frontiersrj.com/journals/ijfetr/
ISSN: 2783-0497 (Online)

(Review Article)

A complete mathematical modelling of robots actuated by epicyclic-gear

Brahim Fernini *

Department of Industrial Engineering, College of Engineering and Computer Science Mustaqbal University, Buraydah 52547 Kingdom of Saudi Arabia.

International Journal of Frontiers in Engineering and Technology Research, 2023, 05(02), 042–060

Publication history: Received on 12 October 2023; revised on 10 December 2023; accepted on 13 December 2023

Article DOI: https://doi.org/10.53294/ijfetr.2023.5.2.0032

Abstract

The industrial robots often use planetary gear system to have high joint torques; therefore, the influence of the rotary inertia of the number of the equally spaced planet-gears on the dynamical behavior of the robot is very important. The main objective of this paper is to develop the dynamic modeling of robot actuated by *(n)* equally spaced planet-gears in the case where the planet-carrier is fixed, no closed solution has been reported for this dynamic modeling, and to compare between the dynamic behavior of robot actuated by *(n+1)* and *(n)* equally spaced planet-gears for a same trajectory planning. The authors derive the explicit dynamic model for an elbow down of 2-R manipulator holding an external mass. Finally, the obtained simulation results of the dynamic modeling are verified by modeling the same robot and using an advanced simulation via *SolidWorks.*

Keywords: Dynamic Modelling; (n) Equally Spaced Planet-Gears; Kinetic Energy; Joint Torque

1. Introduction

Industrial Robots are active systems that require a source of energy to power all their functions. The energy needed for operation must be distributed to the various functions and opportunely modulated, by power converters, which are themselves managed by a suitable low level controller. The power converters provide energy to actuators that transduce the electrical energy supplied by the source into the mechanical energy needed to perform the various tasks [1, 2, 3, 4]. The actuators provide power to act on the mechanical structure against gravity, inertia, and other external forces to modify the pose of the robot's hand [5].

To choose the components of an actuating system, it is worth starting from the requirements imposed on the mechanical power by the force and velocity that describe the joint motion [6].

The execution of joint motions of a manipulator demands low speeds with high torques [6]. In general, such requirements do not allow an effective use of the mechanical features of electrical motors, which typically provide high speeds with low torques in optimal operating conditions. It is then necessary to interpose a transmission (gear) to optimize the transfer of mechanical power from the motor to the joint. During this transfer, the power is dissipated as a result of friction.

The use of gearing is a well-established means for accomplishing such an objective because it is possible to use smaller actuators to deliver larger torques [7].

However, one of the biggest disadvantages of the addition of gearing is the presence of backlash that reduces the effective bandwidth of the controller since high frequency content in the control produces noisy and destructive operation [8].

* Corresponding author: Brahim Fernini

Most industrial robot manipulators are driven by motors through gears with high reductions ratios (from tens to a few hundreds). The use of gears permits an optimization of manipulator static and dynamic performance since the motors can be located on the link preceding the actuated joints along the kinematic chain. Further, typical robot applications require motions with large torques and relatively small velocities, and thus the use of gears allows joint actuation by motors of reduced size [8].

The planetary gear system is widely used in robotics [9, 10], it is composed of one or more outer gears or planet gears, revolving around a sun gear driven by motor. Typically, the planets are mounted on a movable carrier, which rotates relatively to the sun gear. The planetary gearing systems may also include an outer ring gear or annulus, which meshes with the planet gears [9, 10, 11]. A planetary gear example modeled by SolidWorks (2014) is shown in Fig. 1. The advantages of planetary gears over parallel axis gears include high power density, large gear reduction in a small volume, multiple kinematic combinations, pure torsional reactions, and coaxial shafting. Among the disadvantages, one can note high bearing loads, inaccessibility and design complexity [9, 10].

The rotary inertia of the sun gear had a very significant influence on the dynamic behavior of planetary gear train [12]. The effect of gear dynamics and gear ratios on the inverse and forward dynamic models is a problem that is of current interest. For the inverse dynamics, the gear dynamics are most commonly approximated by a diagonal matrix added to the mass matrix [13, 14, 15]. Spong [16] explicitly assumes that the gyroscopic effects of the spinning rotor and gears are negligible. Springer et. Al [17] and Chen [18] used energy methods to add additional terms to the simple rigid model to account for the effects of gear ratios and gyroscopic forces. Walker and Orin [19] developed methods for the fast calculation of the manipulator mass matrix that could include the common gear approximation.

We model the actuator with planetary gear system by using SolidWorks (2020) with no backlash. The planetary gear serves to deliver high output torques. The sun-gear has as input the torque given by the motor and gives torque to the outer ring gear through planet-gears, the output of the outer ring gear connected with the successive link as shown Fig. 1.

In this paper, we are interested on the effects of the rotary inertia of the number of the equally spaced planet-gears on the dynamic of the robot and we will consider that the planet carrier is fixed. A two-revolute (2-R) planar robot is modeled using the SolidWorks as shown in Fig. 1. Motors on each joint coupled with a planetary gear system actuate the manipulator that holds an external mass on the end effector.

The rest of this paper is organized as follows: in the section 2 dynamic model of the manipulator, in section 3 the dynamic simulation, in section 4 discussion of the results followed by a conclusion in section 5.

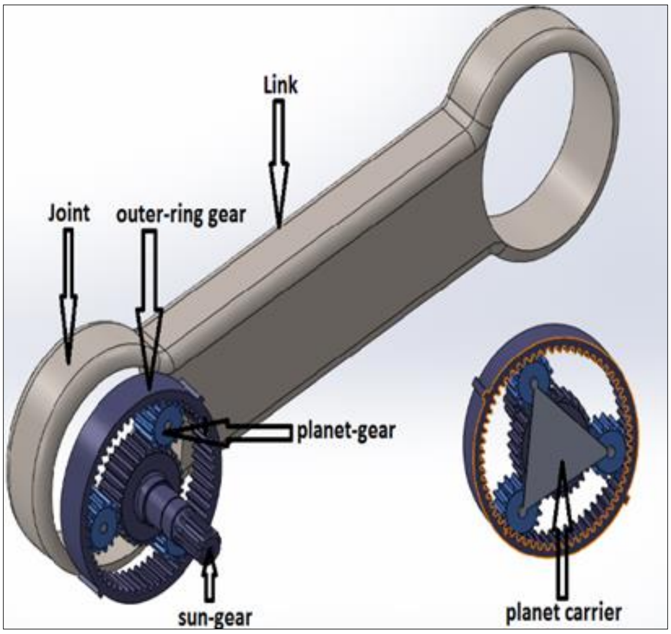

Figure 1 The joint of robot's link actuated by planetary gear system

2. Dynamics modeling of robot actuated by (n) equally spaced planet-gears

2.1. Dynamics modeling of robot actuated by the 3 equally spaced planet-gears

2.1.1. Computation of Kinetic energy of robot

The total kinetic energy is given by the following relation : (See Fig. 2)

$$T=\sum_{i=0}^{n} \left(T_{l_i}+T_{sg_{i+1}}+T_{pg2_{i+1}}+2T_{pg1_{i+1}}\right) \quad \ldots\ldots\ldots\ldots (1)$$

It can be assumed that the contribution in the calculation of the kinetic energy of the (ring-gear) is included in that of the link on which the (ring-gear) is located, and thus the sole contribution of the sun-gear and planet-gears are to be computed. The sun-gears are located on the joint axes, the sun-gear (1) and planet-gears (1) are sitting on the ground and their weight will not affect the dynamics of the manipulators [5, 23].

The total kinetic energy contribution of Link (i) is given by:

$$T=T_{l_i}+T_{sg_{i+1}}+T_{pg2_{i+1}}+2T_{pg1_{i+1}} \quad \ldots\ldots\ldots\ldots (2)$$

$$T_{l_i}=\frac{1}{2} m_{l_i} \dot{p}_{l_i}^T \dot{p}_{l_i}+\frac{1}{2} \omega_i^T I_i \omega_i \quad \ldots\ldots\ldots\ldots (3)$$

$$T_{sg_{i+1}}=\frac{1}{2} m_{sg_{i+1}} \dot{p}_{sg_{i+1}}^T \dot{p}_{sg_{i+1}}+$$

$$+\frac{1}{2} \omega_{sg_{i+1}}^T I_{sg_i} \omega_{sg_{i+1}} \quad \ldots\ldots\ldots\ldots (4)$$

$$T_{pg1_{i+1}}=\frac{1}{2} m_{pg_{i+1}} \dot{p}_{pg_{1i+1}}^T \dot{p}_{pg1_{i+1}}+$$

$$+\frac{1}{2} \omega_{pg_{i+1}}^T I_{pg_i} \omega_{pg_{i+1}} \quad \ldots\ldots\ldots\ldots (5)$$

$$T_{pg2_{i+1}}=\frac{1}{2} m_{pg_{i+1}} \dot{p}_{pg2_{i+1}}^T \dot{p}_{pg2_{i+1}}+$$

$$+\frac{1}{2} \omega_{pg_{i+1}}^T I_{pg_i} \omega_{pg_{i+1}} \quad \ldots\ldots\ldots\ldots (6)$$

$\vartheta_{sg_{i+1}}$ and $\vartheta_{pg_{i+1}}$ are the angular position of the sun-gear and planet-gear respectively.

$$\dot{\vartheta}_{sg_{i+1}}=G_{sgr_{i+1}} \dot{\vartheta}_i \quad \ldots\ldots\ldots\ldots (7)$$

$$\dot{\vartheta}_{pg_{i+1}}=G_{pgr_{i+1}} \dot{\vartheta}_i \quad \ldots\ldots\ldots\ldots (8)$$

here $G_{sgr_{i+1}}$ and $G_{pgr_{i+1}}$ are the gear reduction ratios.

In our case, the planet carrier is fixed, the gear reduction ratios can be calculated by using the relation of Willis [24, 25, 26]:

$$G_{sgr_{i+1}}=G_{sgr_i}=Z_{rg}/Z_{sg}=\vartheta_{sg_i}/\vartheta_i \quad \ldots\ldots\ldots\ldots (9)$$

$$G_{pgr_{i+1}}=G_{pgr_i}=Z_{rg}/Z_{pg}=\vartheta_{pg_i}/\vartheta_i \quad \ldots\ldots\ldots\ldots (10)$$

Where $Z_{sg}$the number of sun-gear teeth is, Z_{rg} is the number of ring-gear teeth, and Z_{pg} is the number of planet-gear teeth. The ring-gear rotates at the rate $G_{sgr_{i+1}}$ in the negative direction of the sun-gear because the sun gear and the ring gear rotate through a planet gear.

The angular velocity composition rule:

The rotation of the link, sun-gear and planet-gears is about the z axis:

$$\omega_{i+1}=\omega_i+\omega_{i,i+1} \quad \ldots\ldots\ldots\ldots (11)$$

According to the angular velocity composition rule and the Eq. 7 the total angular velocity of the sun-gear is:

$$\omega_{sg_{i+1}}=\omega_i+G_{sgr_{i+1}}\dot{\vartheta}_i z_{sg_{i+1}} \quad \ldots\ldots\ldots\ldots (12)$$

Where ω_i is the angular velocity of link (i) on which the sun-gear is located and $z_{sg_{i+1}}$ denotes the unit vector along the sun-gear axis.

$$z_{sg_{i+1}}=z_{sg_i}=z_i=z_0=[0\ 0\ 1]^T \ldots\ldots\ldots\ldots (13)$$

In similar fashion, the angular velocity of the planet gear:

$$\omega_{pg_{i+1}}=\omega_i+G_{pgr_{i+1}}\dot{\vartheta}_i z_{pg_{i+1}} \quad \ldots\ldots\ldots\ldots (14)$$

where:

$$z_{pg_{i+1}}=z_{pg_i}=z_i=z_0=[0\ 0\ 1]^T \quad \ldots\ldots\ldots\ldots (15)$$

Definition of the augmented link (i)

The augmented body (i) is defined as the fictitious body consisting of the particles of body (i) itself and of a mass, equal to that of bodies (i+1,i+2,…n) attached to o_{i+1} . This definition can be applied regardless the type of joint (R) revolute joint or (p) prismatic joint) [28]. By using this definition, in our case, the mass of the augmented link (i) equals to the sum of the mass of link (i) with the mass of sun-gear and planet-gears in joint (i+1) as shown Fig. 2 and the mass of link (i) and the mass of the external mass as shown Fig. 3.

In order to determine such parameters, it is worth associating the kinetic energy contributions of each sun-gear and planet-gears with those of the link on which it is located as shown Fig. 2. Hence, by considering the union of link (i)+sun-gear and planet-gears (i+1) (augmented Link(i)), the kinetic energy of the augmented link can be shown by using the following demonstrations. With reference to the center of mass of the augmented link, the linear velocities of the link, sun-gear and planet-gears can be expressed:

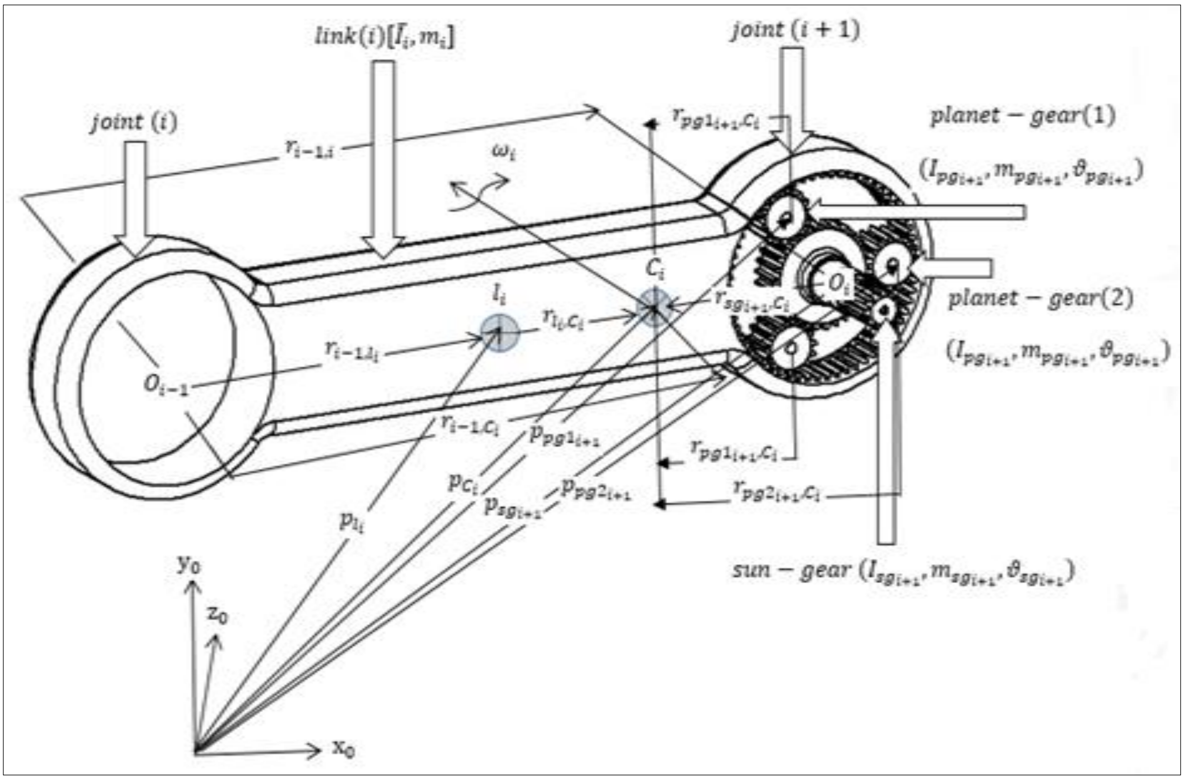

Figure 2 Characterization of augmented link (i) (link+sun-gear+planet-gears) for Kinetic energy

$$\dot{p}_{l_i} = \dot{p}_{C_i} + \omega_i \times r_{C_i,l_i} \quad \ldots\ldots\ldots\ldots (16)$$

$$\dot{p}_{sg_{i+1}} = \dot{p}_{C_i} + \omega_i \times r_{C_i,sg_{i+1}} \quad \ldots\ldots\ldots\ldots (17)$$

$$\dot{p}_{pg1_{i+1}} = \dot{p}_{C_i} + \omega_i \times r_{C_i,pg1_{i+1}} \quad \ldots\ldots\ldots\ldots (18)$$

$$\dot{p}_{pg2_{i+1}} = \dot{p}_{C_i} + \omega_i \times r_{C_i,pg2_{i+1}} \quad \ldots\ldots\ldots\ldots (19)$$

Kinetic energy of link (i) relative to the overall centre of mass C_i

By substituting Eq. 16 into Eq. 3, the kinetic energy of link (i) is given by the following:

$$T_{l_i} = \frac{1}{2} m_{l_i} \dot{p}_{c_i}^T \dot{p}_{c_i} S(\omega_i) m_{l_i} r_{C_i,l_i} + \frac{1}{2} m_{l_i} \omega_i^T S^T(r_{C_i,l_i}) S(r_{C_i,l_i}) \omega_i + \frac{1}{2} \omega_i^T I_i \omega_i \quad \ldots\ldots\ldots\ldots (20)$$

By virtue of Steiner theorem, the following matrix is obtained:

$$\Gamma_{l_i} = I_{l_i} + m_{l_i} S^T(r_{C_i,l_i}) S(r_{C_i,l_i}) \quad \ldots\ldots\ldots\ldots (21)$$

where:

$$I_{l_i} = \begin{bmatrix} I_{l_{ixx}} & I_{l_{ixy}} & I_{l_{ixz}} \\ I_{l_{iyx}} & I_{l_{iyy}} & I_{l_{iyz}} \\ I_{l_{izx}} & I_{l_{izy}} & I_{l_{izz}} \end{bmatrix} \quad \ldots\ldots\ldots\ldots (22)$$

and:

$$m_{l_i} r_{C_i,l_i} = \begin{bmatrix} m_{l_i} r_{C_i,l_{ix}} & m_{l_i} r_{C_i,l_{iy}} & m_{l_i} r_{C_i,l_{iz}} \end{bmatrix}^T \quad \ldots\ldots\ldots\ldots (23)$$

$$S(r_{C_i,l_i}) = \begin{bmatrix} 0 & -r_{C_i,l_{iz}} & r_{C_i,l_{iy}} \\ r_{C_i,l_{iz}} & 0 & -r_{C_i,l_{ix}} \\ -r_{C_i,l_{iy}} & r_{C_i,l_{ix}} & 0 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (24)$$

Γ_{l_i} represents the inertia tensor of the link (i) relative to the overall centre of mass C_i, Eq. 20 can be written as :

$$T_{l_i} = \frac{1}{2} m_{i_i} \dot{p}_{c_i}^T \dot{p}_{c_i} + \dot{p}_{c_i}^T S(\omega_i) m_{l_i} r_{C_i,l_i} + \frac{1}{2} \omega_i^T \Gamma_{l_i} \omega_i \quad \ldots\ldots\ldots\ldots (25)$$

Kinetic energy of sun-gear relative to the overall center of mass C_i

By substituting Eq. 17 into Eq. 4 and exploiting Eq. 12, we obtain:

$$T_{sg_{i+1}} = \frac{1}{2} m_{sg_{i+1}} \dot{p}_{c_i}^T \dot{p}_{c_i} + \dot{p}_{c_i}^T S(\omega_i) m_{sg_{i+1}} r_{C_i,sg_{i+1}} + \frac{1}{2} \omega_i^T \Gamma_{sg_{i+1}} \omega_i + G_{sgr,i+1} \dot{q}_{i+1} z_{sg_{i+1}}^T I_{sg_{i+1}} \omega_i + \frac{1}{2} G_{sgr,i+1}^2 \dot{q}_{i+1}^2 z_{sg_{i+1}}^T I_{sg_{i+1}} z_{sg_{i+1}} \quad \ldots\ldots\ldots\ldots (26)$$

By virtue of Steiner theorem, the following matrix is obtained:

$$\Gamma_{sg_{i+1}} = I_{sg_{i+1}} + m_{sg_{i+1}} S^T(r_{C_i,sg_{i+1}}) S(r_{C_i,sg_{i+1}}) \quad \ldots\ldots\ldots\ldots (27)$$

Where:

$$I_{sg_{i+1}} = \begin{bmatrix} I_{sg_{i+1(xx)}} & I_{sg_{i+1(xy)}} & I_{sg_{i+1(xz)}} \\ I_{sg_{i+1(yx)}} & I_{sg_{i+1(yy)}} & I_{sg_{i+1(yz)}} \\ I_{sg_{i+1(zx)}} & I_{sg_{i+1(zy)}} & I_{sg_{i+1(zz)}} \end{bmatrix} \quad (28)$$

and:

$$m_{sg_{i+1}} r_{C_i,sg_{i+1}} = \begin{bmatrix} m_{sg_{i+1}} r_{C_i,sg_{i+1x}}, & m_{sg_{i+1}} r_{C_i,sg_{i+1y}}, & m_{sg_{i+1}} r_{C_i,sg_{i+1z}} \end{bmatrix}^T \quad \ldots\ldots\ldots\ldots (29)$$

$$S(r_{C_i,sg1_{i+1}}) = \begin{bmatrix} 0 & -r_{C_i,sg_{i+1z}} & r_{C_i,sg_{i+1y}} \\ r_{C_i,sg_{i+1z}} & 0 & -r_{C_i,sg_{i+1x}} \\ -r_{C_i,sg_{i+1y}} & r_{C_i,sg_{i+1x}} & 0 \end{bmatrix} \ldots\ldots\ldots\ldots (30)$$

$\Gamma_{sg_{i+1}}$ represents the inertia tensor of the sun-gear (i+1) relative to the overall center of mass C_i.

Kinetic energy of planet-gear (1) relative to the overall center of mass C_i

By substituting Eq. 18 into Eq. 5 and exploiting Eq. 14, we obtain

$$T_{pg1_{i+1}} = \frac{1}{2} m_{pg_{i+1}} \dot{p}_{c_i}^T \dot{p}_{c_i} + \dot{p}_{c_i}^T S(\omega_i)\, m_{pg_{i+1}} r_{C_i,pg1_{i+1}} + \frac{1}{2} \omega_i^T \Gamma_{pg_{i+1}} \omega_i + G_{pgr,i+1} \dot{q}_{i+1} z_{pg_{i+1}}^T I_{pg_{i+1}} \omega_i + \frac{1}{2} G_{pgr,i+1}^2 \dot{q}_{i+1}^2 z_{pg_{i+1}}^T I_{pg_{i+1}} z_{pg_{i+1}} \ldots\ldots\ldots\ldots (31)$$

By virtue of Steiner theorem, the following matrix is obtained:

$$\Gamma_{pg1_{i+1}} = I_{pg1_{i+1}} + m_{pg_{i+1}} S^T(r_{C_i,pg1_{i+1}}) S(r_{C_i,pg1_{i+1}}) \quad (32)$$

where:

$$I_{pg1_{i+1}} = \begin{bmatrix} I_{pg1_{i+1(xx)}} & I_{pg1_{i+1(xy)}} & I_{pg1_{i+1(xz)}} \\ I_{pg1_{i+1(yx)}} & I_{pg1_{i+1(yy)}} & I_{pg1_{i+1(yz)}} \\ I_{pg1_{i+1(zx)}} & I_{pg1_{i+1(zy)}} & I_{pg1_{i+1(zz)}} \end{bmatrix} \ldots\ldots\ldots\ldots (33)$$

and:

$$m_{pg_{i+1}} r_{C_i,pg1_{i+1}} = \begin{bmatrix} m_{pg_{i+1}} r_{C_i,pg1_{i+1x}} & m_{pg_{i+1}} r_{C_i,pg1_{i+1y}} & m_{pg_{i+1}} r_{C_i,pg1_{i+1z}} \end{bmatrix}^T \ldots\ldots\ldots\ldots (34)$$

$$S(r_{C_i,pg1_{i+1}}) = \begin{bmatrix} 0 & -r_{C_i,pg1_{i+1z}} & r_{C_i,pg1_{i+1y}} \\ r_{C_i,pg1_{i+1z}} & 0 & -r_{C_i,pg1_{i+1x}} \\ -r_{C_i,pg1_{i+1y}} & r_{C_i,pg1_{i+1x}} & 0 \end{bmatrix} \ldots\ldots\ldots\ldots (35)$$

$\Gamma_{pg1_{i+1}}$ represents the inertia tensor of the planet-gear relative to the overall center of mass C_i.

Kinetic energy of planet-gear (2) relative to the overall center of mass C_i

By substituting Eq. 19 into Eq. 6 and exploiting Eq. 14, we obtain

$$T_{pg2_{i+1}} = \frac{1}{2} m_{pg_{i+1}} \dot{p}_{c_i}^T \dot{p}_{c_i} + \dot{p}_{c_i}^T S(\omega_i)\, m_{pg_{i+1}} r_{C_i,pg2_{i+1}} + \frac{1}{2} \omega_i^T \Gamma_{pg2_{i+1}} \omega_i + G_{pgr,i+1} \dot{q}_{i+1} z_{pg_{i+1}}^T I_{pg2_{i+1}} \omega_i + \frac{1}{2} G_{pgr,i+1}^2 \dot{q}_{i+1}^2 z_{pg_{i+1}}^T I_{pg2_{i+1}} z_{pg_{i+1}} \ldots\ldots\ldots\ldots (36)$$

By virtue of Steiner theorem, the following matrix is obtained:

$$\Gamma_{pg2_{i+1}} = I_{pg2_{i+1}} + m_{pg_{i+1}} S^T(r_{C_i,pg2_{i+1}}) S(r_{C_i,pg2_{i+1}}) \ldots\ldots\ldots\ldots (37)$$

where:

$$I_{pg2_{i+1}} = \begin{bmatrix} I_{pg2_{i+1(xx)}} & I_{pg2_{i+1(xy)}} & I_{pg2_{i+1(xz)}} \\ I_{pg2_{i+1(yx)}} & I_{pg2_{i+1(yy)}} & I_{pg2_{i+1(yz)}} \\ I_{pg2_{i+1(zx)}} & I_{pg2_{i+1(zy)}} & I_{pg2_{i+1(zz)}} \end{bmatrix} \ldots\ldots\ldots\ldots (38)$$

and:

$$m_{pg_{i+1}} r_{C_i,pg2_{i+1}} = \begin{bmatrix} m_{pg_{i+1}} r_{C_i,pg2_{i+1x}} & m_{pg_{i+1}} r_{C_i,pg2_{i+1y}} & m_{pg_{i+1}} r_{C_i,pg2_{i+1z}} \end{bmatrix}^T \ldots\ldots\ldots\ldots (39)$$

$$S(r_{C_i,pg2_{i+1}})=\begin{bmatrix}0 & -r_{C_i,pg2_{i+1z}} & r_{C_i,pg2_{i+1y}} \\ r_{C_i,pg2_{i+1z}} & 0 & -r_{C_i,pg2_{i+1x}} \\ -r_{C_i,pg2_{i+1y}} & r_{C_i,pg2_{i+1x}} & 0\end{bmatrix} \quad \ldots\ldots\ldots\ldots (40)$$

$\Gamma_{pg2_{i+1}}$ represents the inertia tensor of the planet-gear relative to the overall center of mass C_i.

Summing the contributions in Eq. 25, 26, 31 and Eq. 36 as in Eq. 2 gives the expression of the kinetic energy of augmented link (i) in the form :

$$T_i=\frac{1}{2} m_i \dot{p}_{c_i}^T \dot{p}_{c_i}+\frac{1}{2} \omega_i^T \Gamma_i \omega_i+G_{sgr,i+1} \dot{q}_{i+1} z_{sg_{i+1}}^T I_{sg_{i+1}} \omega_i+\frac{1}{2} G_{sgr,i+1}^2 \dot{q}_{i+1}^2 z_{sg_{i+1}}^T I_{sg_{i+1}} z_{sg_{i+1}}+G_{pgr,i+1} \dot{q}_{i+1} z_{pg_{i+1}}^T I_{pg2_{i+1}} \omega_i+\frac{1}{2} G_{pg2r,i+1}^2 \dot{q}_{i+1}^2 z_{pg2_{i+1}}^T I_{pg2_{i+1}} z_{sg_{i+1}}+2G_{pgr,i+1} \dot{q}_{i+1} z_{pg2_{i+1}}^T I_{pg1_{i+1}} \omega_i+G_{pgr,i+1}^2 \dot{q}_{i+1}^2 z_{pg1_{i+1}}^T I_{pg1_{i+1}} z_{pg1_{i+1}} \quad \ldots\ldots\ldots\ldots (41)$$

where $m_i=m_{l_i}+m_{sg_{i+1}}+3m_{pg_{i+1}}$ and $\Gamma_i=\Gamma_{l_i}+\Gamma_{sg_{i+1}}+2\Gamma_{pg1_{i+1}}+\Gamma_{pg2_{i+1}}$ are respectively the overall mass and inertia tensor relative to the position of center of mass of the augmented link.

The inertia tensor of the augmented link can be written as:

$$\Gamma_i=\begin{bmatrix}\Gamma_{ix} & -\Gamma_{ixy} & -\Gamma_{ixz} \\ -\Gamma_{iyx} & \Gamma_{iyy} & -\Gamma_{iyz} \\ -\Gamma_{izx} & -\Gamma_{izy} & \Gamma_{izz}\end{bmatrix} \quad \ldots\ldots\ldots\ldots (42)$$

The position of center of mass of augmented link can be expressed by:

$$m_{l_i} P_{l_i}+m_{sg_{i+1}} P_{sg_{i+1}}+2m_{pg_{i+1}} P_{pg1_{i+1}}+m_{pg_{i+1}} P_{pg2_{i+1}}=m_i P_{C_i} \quad \ldots\ldots\ldots\ldots (43)$$

Computing the inertia tensor of the augmented link (link+external mass) relative to the position of center of mass as shown Fig. 3:

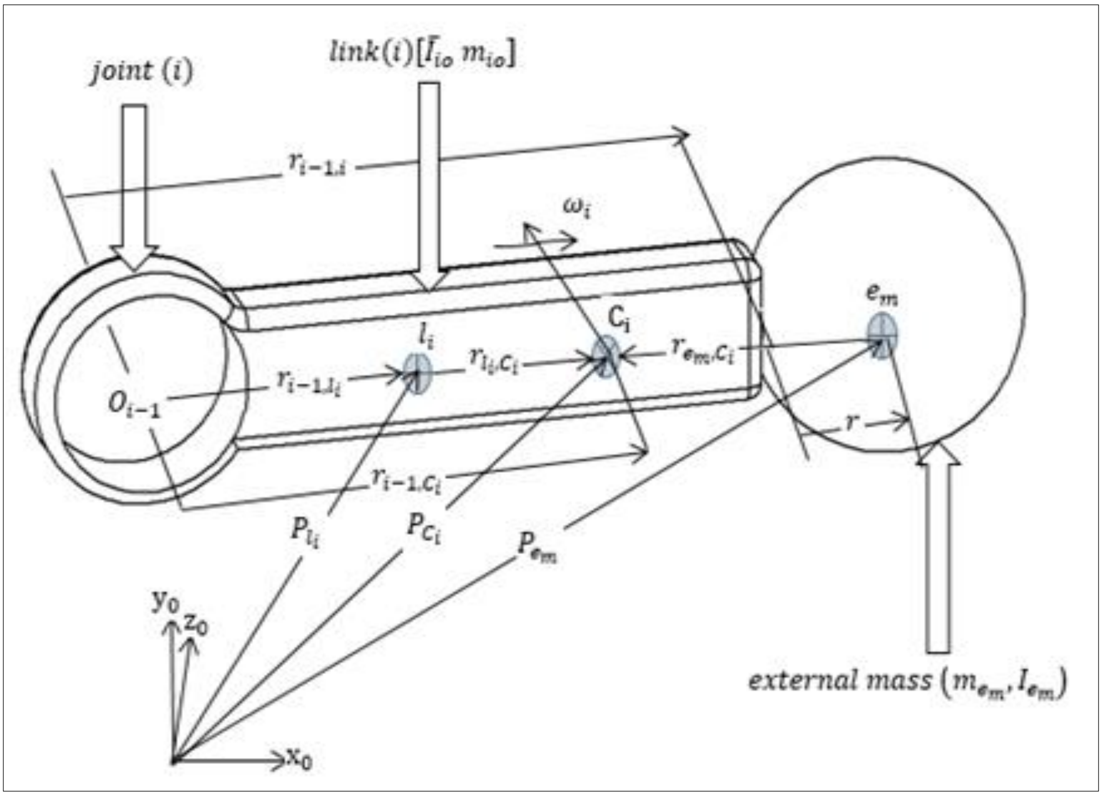

Figure 3 Characterization of augmented link (i) (link+external mass) for Kinetic energy

The external mass is a sphere with mass (m_{e_m}) and radius (r).The inertia tensor about its center of mass is :

$$I_{e_m}=\begin{bmatrix} I_{e_{mxx}} & I_{e_{mxy}} & I_{e_{mxz}} \\ I_{e_{myx}} & I_{e_{myy}} & I_{e_{myz}} \\ I_{e_{mzx}} & I_{e_{mzy}} & I_{e_{mzz}} \end{bmatrix} \quad (44)$$

$$m_{e_m} r_{C_i,e_m}=[m_{e_m} r_{C_i,e_{mx}}, m_{e_m} r_{C_i,e_{my}}, m_{e_m} r_{C_i,e_{mz}}]^T \; \ldots\ldots\ldots\ldots (45)$$

$$(r_{C_i,e_m})=\begin{bmatrix} 0 & -r_{C_i,e_{mz}} & r_{C_i,e_{my}} \\ r_{C_i,e_{mz}} & 0 & -r_{C_i,e_{mx}} \\ -r_{C_i,e_{my}} & r_{C_i,e_{mx}} & 0 \end{bmatrix} \quad (46)$$

$$\Gamma_{e_m}=I_{e_m}+m_{e_m} S^T(r_{C_i,e_m}) S(r_{C_i,e_m}) \; \ldots\ldots\ldots\ldots (47)$$

where Γ_{e_m} represents the inertia tensor of the external mass relative to the overall centre of mass C_i. .

The position of center of mass of augmented link can be expressed by:

$$m_{l_i} p_{l_i}+m_{e_m} p_{e_m}=m_{i0} p_{C_i} \; \ldots\ldots\ldots\ldots (48)$$

The kinetic energy of the augmented link can be expressed by:

$$T_i=\frac{1}{2} m_{i0} \dot{p}_{c_i}^T \dot{p}_{c_i}+\frac{1}{2} \omega_i^T \Gamma_{io} \omega_i \; \ldots\ldots\ldots\ldots (49)$$

where $m_{io}=m_{l_i}+m_{e_m}$ and $\Gamma_{io}=\Gamma_{l_i}+\Gamma_{e_m}$ are respectively the overall mass and inertia tensor relative to the position of center of mass of the augmented link.

On the assumption that the link, sun-gear, planet-gears and external mass have a symmetric mass distribution about the axis of rotation and the axis of link (i) frame coincides with the central axis of

inertia, then the inertia products are null and the inertia tensor relative to the center of mass is a diagonal matrix [5, 23].

The inertia tensor of sun-gear about its center of mass:

$$I_{sg_{i+1}}^{sg_{i+1}}=\begin{bmatrix} I_{sg_{i+1(xx)}}^{sg_{i+1}} & 0 & 0 \\ 0 & I_{sg_{i+1(yy)}}^{sg_{i+1}} & 0 \\ 0 & 0 & I_{sg_{i+1(zz)}}^{sg_{i+1}} \end{bmatrix} \; \ldots\ldots\ldots\ldots (50)$$

The inertia tensor of planet-gear about its center of mass:

$$I_{pg_{i+1}}^{pg_{i+1}}=\begin{bmatrix} I_{pg_{i+1(xx)}}^{pg_{i+1}} & 0 & 0 \\ 0 & I_{pg_{i+1(yy)}}^{pg_{i+1}} & 0 \\ 0 & 0 & I_{pg_{i+1(zz)}}^{pg_{i+1}} \end{bmatrix} \; \ldots\ldots\ldots\ldots (51)$$

Since the aim is to determine a set of dynamic parameters independent of the manipulator joint configuration, it is worth referring the inertia tensor of the link Γ_i to frame R_i attached to the link and the inertia tensor of sun-gear and planet-gear $I_{sg_{i+1}}$, $I_{pg_{i+1}}$ to frame $R_{sg_{i+1}}$ and $R_{pg_{i+1}}$ respectively,so that it is diagonal. In view of Eq. 50 and Eq. 51 one has:

$$I_{sg_{i+1}} z_{sg_{i+1}}=R_{sg_{i+1}} I_{sg_{i+1}}^{sg_{i+1}} R_{sg_{i+1}}^T z_{sg_{i+1}}=I_{sg_{i+1}} z_{sg_{i+1}} \; \ldots\ldots\ldots\ldots (52)$$

$$I_{pg_{i+1}} z_{pg_{i+1}}=R_{pg_{i+1}} I_{pg_{i+1}}^{pg_{i+1}} R_{pg_{i+1}}^T z_{pg_{i+1}}=I_{pg_{i+1}} z_{pg_{i+1}} \; \ldots\ldots\ldots\ldots (53)$$

where $I_{sg_{i+1}}=I_{sg_{i+1(zz)}}$ and $I_{pg_{i+1}}=I_{pg_{i+1(zz)}}=I_{pg1_{i+1(zz)}}=I_{pg2_{i+1(zz)}}$ denote the constant scalar moment of inertia of the sun-gear and planet-gear about the rotation axis.

Therefore, the kinetic energy shown in Eq. 41 becomes:

$$T_i=\frac{1}{2} m_i \dot{p}_{c_i}^{iT} \dot{p}_{c_i}^i+\frac{1}{2} \omega_i^T \Gamma_i^i \omega_i+G_{sgr,i+1} \dot{q}_{i+1} I_{sg_{i+1}} z_{sg_{i+1}}^{iT} \omega_i+\frac{1}{2} G_{sgr,i+1}^2 \dot{q}_{i+1}^2 I_{sg_{i+1}}+3G_{pgr,i+1} \dot{q}_{i+1} I_{pg_{i+1}} z_{pg_{i+1}}^{iT} \omega_i+\frac{3}{2} G_{pgr,i+1}^2 \dot{q}_{i+1}^2 I_{pg_{i+1}} \quad \ldots\ldots\ldots\ldots (54)$$

According to the linear velocity composition rule for link (i), one may write:

$$\dot{p}_{c_i}^i=\dot{p}_i^i+\omega_i^i\times r_{i,C_i}^i \quad \ldots\ldots\ldots\ldots (55)$$

where all the vectors have been referred to frame (i), note that $r_{i,C_i}^i$is fixed in such a frame. Substituting Eq. 55 in Eq. 54 gives:

$$T_i=\frac{1}{2} m_i \dot{p}_i^{iT} \dot{p}_i^i+\dot{p}_i^{iT} S(\omega_i^i) m_i r_{i,C_i}^i+\frac{1}{2} \omega_i^{iT} \bar{\Gamma}_i^i \omega_i^i+G_{sgr,i+1} \dot{q}_{i+1} I_{sg_{i+1}} z_{sg_{i+1}}^{iT} \omega_i+\frac{1}{2} G_{sgr,i+1}^2 \dot{q}_{i+1}^2 I_{sg_{i+1}}+3G_{pgr,i+1} \dot{q}_{i+1} I_{pg_{i+1}} z_{pg_{i+1}}^{iT} \omega_i+\frac{3}{2} G_{pgr,i+1}^2 \dot{q}_{i+1}^2 I_{pg_{i+1}} \quad \ldots\ldots\ldots\ldots (56)$$

In similar fashion, the kinetic energy of the augmented link (Link+sun-gear+planet-gears) actuated by (n) equally planet-gears:

$$T_i=\frac{1}{2} m_i \dot{p}_i^{iT} \dot{p}_i^i+\dot{p}_i^{iT} S(\omega_i^i) m_i r_{i,C_i}^i+\frac{1}{2} \omega_i^{iT} \bar{\Gamma}_i^i \omega_i^i+G_{sgr,i+1} \dot{q}_{i+1} I_{sg_{i+1}} z_{sg_{i+1}}^{iT} \omega_i+\frac{1}{2} G_{sgr,i+1}^2 \dot{q}_{i+1}^2 I_{sg_{i+1}}+(n)G_{pgr,i+1} \dot{q}_{i+1} I_{pg_{i+1}} z_{pg_{i+1}}^{iT} \omega_i+\frac{(n)}{2} G_{pgr,i+1}^2 \dot{q}_{i+1}^2 I_{pg_{i+1}} \quad \ldots\ldots\ldots\ldots (57)$$

where

$$\bar{\Gamma}_i^i=\Gamma_i^i+m_i S^T(r_{i,C_i}^i) S(r_{i,C_i}^i) \quad \ldots\ldots\ldots\ldots (58)$$

represents the inertia tensor with respect to the origin of frame (i), according to Steiner theorem. Let:

$$r_{i,C_i}^i=[r_{i,C_{ix}}^i \;\; r_{i,C_{iy}}^i \;\; r_{i,C_{iz}}^i]^T \quad \ldots\ldots\ldots\ldots (59)$$

The first moment of inertia is:

$$m_i r_{i,C_i}^i=[m_i r_{i,C_{ix}}^i \;\; m_i r_{i,C_{iy}}^i \;\; m_i r_{i,C_{iz}}^i]^T \quad \ldots\ldots\ldots\ldots (60)$$

From Eq. 58 the inertia tensor of augmented link (i) is:

$$\bar{\Gamma}_i^i=\begin{bmatrix} \Gamma_{ixx}^i+m_i (r_{s_{i+1},C_{iy}}^{i2}+r_{s_{i+1},C_{iz}}^{i2}) & -\Gamma_{ixy}^i m_i r_{s_{i+1},C_{ix}}^i r_{s_{i+1},C_{iy}}^i & -\Gamma_{ixz}^i m_i r_{s_{i+1},C_{ix}}^i r_{s_{i+1},C_{iz}}^i \\ -\Gamma_{ixy}^i m_i r_{s_{i+1},C_{ix}}^i r_{s_{i+1},C_{iy}}^i & \Gamma_{iyy}^i+m_i (r_{s_{i+1},C_{ix}}^{i2}+r_{s_{i+1},C_{iz}}^{i2}) & -\Gamma_{iyz}^i m_i r_{s_{i+1},C_{iy}}^i r_{s_{i+1},C_{iz}}^i \\ -\Gamma_{ixz}^i m_i r_{s_{i+1},C_{ix}}^i r_{s_{i+1},C_{iz}}^i & -\Gamma_{iyz}^i m_i r_{s_{i+1},C_{iy}}^i r_{s_{i+1},C_{iz}}^i & \Gamma_{izz}^i+m_i (r_{s_{i+1},C_{ix}}^{i2}+r_{s_{i+1},C_{iy}}^{i2}) \end{bmatrix} \quad \ldots\ldots\ldots\ldots (61)$$

$$\bar{\Gamma}_i^i=\begin{bmatrix} \bar{\Gamma}_{ixx}^i & -\bar{\Gamma}_{ixy}^i & -\bar{\Gamma}_{ixz}^i \\ -\bar{\Gamma}_{ixy}^i & \bar{\Gamma}_{iyy}^i & -\bar{\Gamma}_{iyz}^i \\ -\bar{\Gamma}_{izx}^i & -\bar{\Gamma}_{izy}^i & \bar{\Gamma}_{izz}^i \end{bmatrix} \quad \ldots\ldots\ldots\ldots (62)$$

In similar fashion, the kinetic energy of the augmented link (link+external mass) actuated by (n) equally spaced planet-gears:

$$T_i=\frac{1}{2} m_{io} \dot{p}_i^{iT} \dot{p}_i^i+\dot{p}_i^{iT} S(\omega_i^i) m_{io} r_{e_m,C_i}^i+\frac{1}{2} \omega_i^{iT} \bar{\Gamma}_{io}^{io} \omega_i^i++G_{sgr,i+1} \dot{q}_{i+1} I_{sg_{i+1}} z_{sg_{i+1}}^{iT} \omega_i+\frac{1}{2} G_{sgr,i+1}^2 \dot{q}_{i+1}^2 I_{sg_{i+1}}+(n)G_{pgr,i+1} \dot{q}_{i+1} I_{pg_{i+1}} z_{pg_{i+1}}^{iT} \omega_i+\frac{(n)}{2} G_{pgr,i+1}^2 \dot{q}_{i+1}^2 I_{pg_{i+1}} \quad \ldots\ldots\ldots\ldots (63)$$

where

$$\bar{\Gamma}_{io}^{io}=\Gamma_{io}^{io}+m_{io} S^T(r_{e_m,C_i}^i) S(r_{e_m,C_i}^i) \quad \ldots\ldots\ldots\ldots (64)$$

represents the inertia tensor with respect to the origin of frame (i), according to Steiner theorem. Let:

$$r_{(e_m,C_i)}^i=[r_{(e_m,C_{ix})}^i, r_{(e_m,C_{iy})}^i, r_{(e_m,C_{iz})}^i]^T \quad \ldots\ldots\ldots\ldots (65)$$

The first moment of inertia is:

$$m_{io} r_{(e_m,C_i)}^i=[m_{io} r_{(e_m,C_{ix})}^i \;\; m_{io} r_{(e_m,C_{iy})}^i \;\; m_{io} r_{(e_m,C_{iz})}^i]^T \quad \ldots\ldots\ldots\ldots (66)$$

$$\bar{\Gamma}_{io}^{io}=\begin{bmatrix} \Gamma_{ioxx}^{io}+m_{io}(r_{(e_m,C_{iy})}^{i^2}+r_{(e_m,C_{iz})}^{i^2}) & -\Gamma_{ioxy}^i m_{io} r_{(e_m,C_{ix})}^i r_{(e_m,C_{iy})}^i & -\Gamma_{ioxz}^i m_{io} r_{(e_m,C_{ix})}^i r_{(e_m,C_{iz})}^i \\ -\Gamma_{ioxy}^i m_{io} r_{(e_m,C_{ix})}^i r_{(e_m,C_{iy})}^i & \Gamma_{ioyy}^{io}+m_{io}(r_{(e_m,C_{ix})}^{i^2}+r_{(e_m,C_{iz})}^{i^2}) & -\Gamma_{ioyz}^i m_{io} r_{(e_m,C_{iy})}^i r_{(e_m,C_{iz})}^i \\ -\Gamma_{ioxz}^i m_{io} r_{(e_m,C_{ix})}^i r_{(e_m,C_{iz})}^i & -\Gamma_{ioyz}^i m_{io} r_{(e_m,C_{iy})}^i r_{(e_m,C_{iz})}^i & \Gamma_{iozz}^{io}+m_{io}(r_{(e_m,C_{ix})}^{i^2}+r_{(e_m,C_{iy})}^{i^2}) \end{bmatrix} \quad \ldots\ldots\ldots\ldots (67)$$

$$\bar{\Gamma}_{io}^{io}=\begin{bmatrix} \bar{\Gamma}_{ioxx}^{io} & -\bar{\Gamma}_{ioxy}^{io} & -\bar{\Gamma}_{ioxz}^{io} \\ -\bar{\Gamma}_{ioxy}^{io} & \bar{\Gamma}_{ioyy}^{io} & -\bar{\Gamma}_{ioyz}^{io} \\ -\bar{\Gamma}_{iozx}^{io} & -\bar{\Gamma}_{iozy}^{io} & \bar{\Gamma}_{iozz}^{io} \end{bmatrix} \quad \ldots\ldots\ldots\ldots (68)$$

2.1.2. Newton–Euler Formulation

The Newton–Euler formulation describes the motion of the link in terms of a balance of forces and moments acting on it [27, 29], as shown Fig. 4.

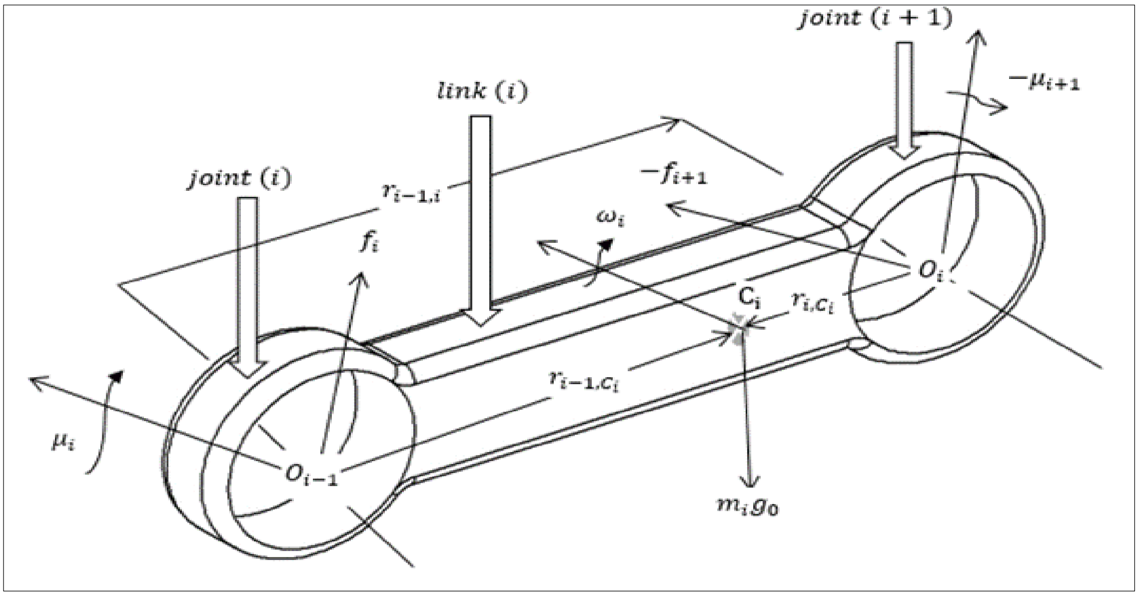

Figure 4 Characterization of augmented link (i) for Newton-Euler formulation

The Newton equation for the translational motion of the center of mass can be written as:

$$f_i-f_{(i-1)}+m_i g_0=m_i \ddot{p}_{(c_i)} \quad \ldots\ldots\ldots\ldots (69)$$

The Euler equation for the rotational motion of the link (referring moments to the center of mass) can be written as:

$$\mu_i+f_i\times r_{(i-1,C_i)}-\mu_{(i+1)}-f_{(i+1)}\times r_{(i,C_i)}=\frac{d}{dt}\left(\Gamma_i \omega_i+G_{(sgr,i+1)} \dot{q}_{(i+1)} I_{(sg_{(i+1)})} z_{(sg_{(i+1)})}+3G_{(pgr,i+1)} \dot{q}_{(i+1)} I_{(pg_{(i+1)})} z_{(pg_{(i+1)})}\right) \quad \ldots\ldots\ldots\ldots (70)$$

Where $G_{(sgr,i+1)} \dot{q}_{(i+1)} I_{(sg_{(i+1)})} z_{(sg_{(i+1)})}$ and $G_{(pgr,i+1)} \dot{q}_{(i+1)} I_{(pg_{(i+1)})} z_{(pg_{(i+1)})}$ are the angular momentum of the sun-gear and planet-gear respectively. Notice that the gravitational force $m_i g_0$ does not generate any moment, since it is concentrated at the centre of mass.

In our case, the angular momentum of the carrier equals to zero, because the carrier is fixed.

The inertia tensor of the augmented link (i) in the base: $\Gamma_i=R_i \bar{I}_i^i R_i^T$ where R_i represents the rotation matrix from frame (i) to the base frame. Substituting this relation in the first term on the right-hand side of Eq. 70 yields:

$$\frac{d}{dt}(\Gamma_i \omega_i)=\dot{R}_i \bar{I}_i^i R_i^T \omega_i+R_i \bar{I}_i^i \dot{R}_i^T \omega_i+R_i \bar{I}_i^i R_i^T \dot{\omega}_i=S(\omega_i) R_i \bar{I}_i^i R_i^T \omega_i+R_i \bar{I}_i^i R_i^T S^T(\omega_i) \omega_i+R_i \bar{I}_i^i R_i^T \dot{\omega}_i=\Gamma_i \dot{\omega}_i+\omega_i\times(\Gamma_i \omega_i) \quad \ldots\ldots\ldots\ldots (71)$$

where:

$$\omega_i=[\omega_{ix}, \omega_{iy}, \omega_{iz}]^T \quad \ldots\ldots\ldots\ldots (72)$$

In our case, the rotation is done only in (z)axis, Eq. 72 becomes:

$$\omega_i=[0\ 0\ \omega_{iz}]^T \quad \ldots\ldots\ldots\ldots (73)$$

and:

$$S(\omega_i)=\begin{bmatrix} 0 & -\omega_{iz} & 0 \\ \omega_{iz} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (74)$$

The second term of Eq. 71 represents the gyroscopic torque induced by the dependence of Γ_i on link orientation. Moreover, by observing that the unit vector $z_{sg_{i+1}}$ rotates accordingly to link(i), the derivative needed in the second term on the right-hand side of Eq. 70 is:

$$\frac{d}{dt}\left(G_{sgr,i+1} \dot{q}_{i+1} I_{sg_{i+1}} z_{sg_{i+1}}+G_{sg,i+1} \dot{q}_{i+1} I_{pg_{i+1}} z_{pg_{i+1}}\right)=G_{sgr,i+1}\left(\ddot{q}_{i+1} I_{sg_{i+1}} z_{sg_{i+1}}+\dot{q}_{i+1} I_{sg_{i+1}} \omega_i\times z_{sg_{i+1}}\right)+G_{pgr,i+1}\left(\ddot{q}_{i+1} I_{pg_{i+1}} z_{pg_{i+1}}+\dot{q}_{i+1} I_{pg_{i+1}} \omega_i\times z_{pg_{i+1}}\right) \quad \ldots\ldots\ldots\ldots (75)$$

In deriving Eq.71, the operator S has been introduced to compute the derivative of R_i, also the property $S^T(\omega_i)\omega_i=0$ has been utilized.

By substituting Eq. 71, 75 in Eq. 70, the resulting Euler equation is:

$$\mu_i+f_i\times r_{i-1,C_i}-\mu_{i+1}-f_{i+1}\times r_{i,C_i}=\Gamma_i \dot{\omega}_i+\omega_i\times(\Gamma_i \omega_i)+G_{sgr,i+1}\left(\ddot{q}_{i+1} I_{sg_{i+1}} z_{sg_{i+1}}+\dot{q}_{i+1} I_{sg_{i+1}} \omega_i\times z_{sg_{i+1}}\right)+3G_{pgr,i+1}\left(\ddot{q}_{i+1} I_{pg_{i+1}} z_{pg_{i+1}}+\dot{q}_{i+1} I_{pg_{i+1}} \omega_i\times z_{pg_{i+1}}\right) \quad \ldots\ldots\ldots\ldots (76)$$

The generalized force at Joint (i) can be computed by projecting the moment μ_i for a revolute joint, along the joint axis. In addition, there is the contribution of the inertia torques $G_{sgr,i} I_{sg_i} \dot{\omega}_{sg_i}^T z_{sg_i}$ and $G_{pgr,i} I_{pg_i} \dot{\omega}_{pg_i}^T z_{pg_i}$ of the sun-gear and planet-gears respectively.

he torque exerted at Joint (i) is expressed by:

$$\tau_i=\mu_i^T z_{i-1}+G_{sgr,i} I_{sg_i} \dot{\omega}_{sg_i}^T z_{sg_i}+3G_{pgr,i} I_{pg_i} \dot{\omega}_{pg_i}^T z_{pg_i}+F_{vi} \dot{\vartheta}_i+F_{si}\,\mathrm{sgn}(\dot{\vartheta}_i) \quad \ldots\ldots\ldots\ldots (77)$$

Where joint viscous torque (F_{vi}) and F_{si} Coulomb friction torque have been included, and in our case, we consider them neglected.

The N-E recursive

The following equations are calculated in current frame, because the recursion is computationally more efficient if all vectors are referred to the current frame on link (i) [27].

- The initial condition

$$\omega_0^0=\dot{\omega}_0^0=\dot{p}_0^0=0 \quad \ldots\ldots\ldots\ldots (78)$$

$$\ddot{p}_0^0=[\,0\ g\ 0\,]^{T} \quad \ldots\ldots\ldots\ldots (79)$$

- Forward recursive equations in current frame

$$\omega_i^i=R_i^{i-1\,T}\left(\omega_{i-1}^{i-1}+\dot{\vartheta}_i z_0\right) \quad \ldots\ldots\ldots\ldots (80)$$

$$\dot{\omega}_i^i=R_i^{i-1\,T}\left(\dot{\omega}_{i-1}^{i-1}+\ddot{\vartheta}_i z_{i-1}+\dot{\vartheta}_i\, \omega_{i-1}^{i-1}\times z_0\right) \quad \ldots\ldots\ldots\ldots (81)$$

$$\dot{\omega}_{sg_i}^{i-1}=\left(\dot{\omega}_{i-1}^{i-1}+G_{sgri}\,\ddot{q}_i\, z_{sg_i}^{i-1}+G_{sgri}\,\dot{q}_i\, \omega_{i-1}^{i-1}\times z_{sg_i}^{i-1}\right) \quad \ldots\ldots\ldots\ldots (82)$$

$$\dot{\omega}_{pg_i}^{i-1}=\left(\dot{\omega}_{i-1}^{i-1}+G_{pgr,i}\,\ddot{q}_i\, z_{g_i}^{i-1}+G_{gri}\,\dot{q}_i\, \omega_{i-1}^{i-1}\times z_{g_i}^{i-1}\right) \quad \ldots\ldots\ldots\ldots (83)$$

$$\ddot{p}_i^i=R_i^{i-1\,T}\,\ddot{p}_{i-1}^{i-1}+\dot{\omega}_i^i\times r_{i-1,i}^i+\omega_i^i\times\left(\omega_i^i\times r_{i-1,i}^i\right) \quad \ldots\ldots\ldots\ldots (84)$$

$$\ddot{p}_{c_i}^i=\ddot{p}_i^i+\dot{\omega}_i^i\times r_{i,C_i}^i+\omega_i^i\times\left(\omega_i^i\times r_{i,C_i}^i\right) \quad \ldots\ldots\ldots\ldots (85)$$

- Backward recursive equations in current frame

$$f_i^i=R_{i+1}^i f_{i+1}^{i+1}+m_i\,\ddot{p}_{c_i}^i \quad (86)$$

$$\mu_i^i=-f_i^i\times\left(r_{i-1,i}^i+r_{i,C_i}^i\right)+R_{i+1}^i\mu_{i+1}^{i+1}+R_{i+1}^i f_{i+1}^{i+1}\times r_{i,C_i}^i+\bar{I}_i^i\,\dot{\omega}_i^i+\omega_i^i\times\left(\bar{I}_i^i\,\omega_i^i\right)+G_{sgr,i+1}\left(\ddot{q}_{i+1} I_{sg_{i+1}} z_{sg_{i+1}}^i+\dot{q}_{i+1} I_{sg_{i+1}}\,\omega_i^i\times z_{sg_{i+1}}^i\right)+3G_{pgr,i+1}\left(\ddot{q}_{i+1} I_{pg_{i+1}} z_{pg_{i+1}}^i+\dot{q}_{i+1} I_{pg_{i+1}}\,\omega_i^i\times z_{pg_{i+1}}^i\right) \quad \ldots\ldots\ldots\ldots (87)$$

The joint exerted at the joint (i) in current frame

$$\tau_i^i=\mu_i^{i\,T} R_i^{i-1\,T} z_0+G_{sgr,i}\, I_{sg_i}\,\dot{\omega}_{sg_i}^{i-1\,T} z_{sg_i}^{i-1}+3G_{pgr,i}\, I_{pg_i}\,\dot{\omega}_{pg_i}^{i-1\,T} z_{pg_i}^{i-1} \quad \ldots\ldots\ldots\ldots (88)$$

In similar fashion, the equation of the joint torque of robot's link actuated by (n) equally planet-gears:

$$\tau_i^i=\mu_i^{i\,T} R_i^{i-1\,T} z_0+G_{sgr,i}\, I_{sg_i}\,\dot{\omega}_{sg_i}^{i-1\,T} z_{sg_i}^{i-1}+(n)G_{pgr,i}\, I_{pg_i}\,\dot{\omega}_{pg_i}^{i-1\,T} z_{pg_i}^{i-1} \quad \ldots\ldots\ldots\ldots (89)$$

With $\mu_i^i=-f_i^i\times\left(r_{i-1,i}^i+r_{i,C_i}^i\right)+R_{i+1}^i\mu_{i+1}^{i+1}+R_{i+1}^i f_{i+1}^{i+1}\times r_{i,C_i}^i+\bar{I}_i^i\,\dot{\omega}_i^i+\omega_i^i\times\left(\bar{I}_i^i\,\omega_i^i\right)+G_{sgr,i+1}\left(\ddot{q}_{i+1} I_{sg_{i+1}} z_{sg_{i+1}}^i+\dot{q}_{i+1} I_{sg_{i+1}}\,\omega_i^i\times z_{sg_{i+1}}^i\right)+(n)G_{pgr,i+1}\left(\ddot{q}_{i+1} I_{pg_{i+1}} z_{pg_{i+1}}^i+\dot{q}_{i+1} I_{pg_{i+1}}\,\omega_i^i\times z_{pg_{i+1}}^i\right) \quad \ldots\ldots\ldots\ldots (90)$

3. Dynamic Simulation

As an example of simulation, we choose an elbow down of 2-R planar robot as shown Fig. 5. The links of robot have the same length (0.4m) and actuated by 3, 4, 5 and 6 equally spaced planet-gears as shown Fig. 6. The simulation of the kinetic energy and the joint torque can be done by using the mass properties shown in Table. 1. The links of robot, sun-gear, planet-gears and external mass are designed by using SolidWorks (2014) in such way the inertia tensor of link, sun-gear, planet-gears and external mass has a diagonal matrix about its center of mass in current frame, then the inertia tensor of the augmented link is also diagonal matrix about its overall center of mass C_i in current frame.

The 2-R robot moves between two positions during 1s (x=0.8m, y=0m) to (x=0.4499m, y=0.5864m) with jerk zero at start-stop path and gives the trajectory shown in Fig (7) and it's verified by Matlab/simulink. Generation of trajectories with a bounded value of jerk improves the tracking accuracy and will allow to reach a higher speed of task execution, with eventually a reduction in the excitation of the resonant frequencies and the vibrations caused by the planetary gear system [30, 31, 32, 33]. We can compare between the two postures of 2-R robot by using the equations of the serial planar manipulators [34] in the dynamics modeling shown in section. 2 to choose the posture that has the power saving [35].

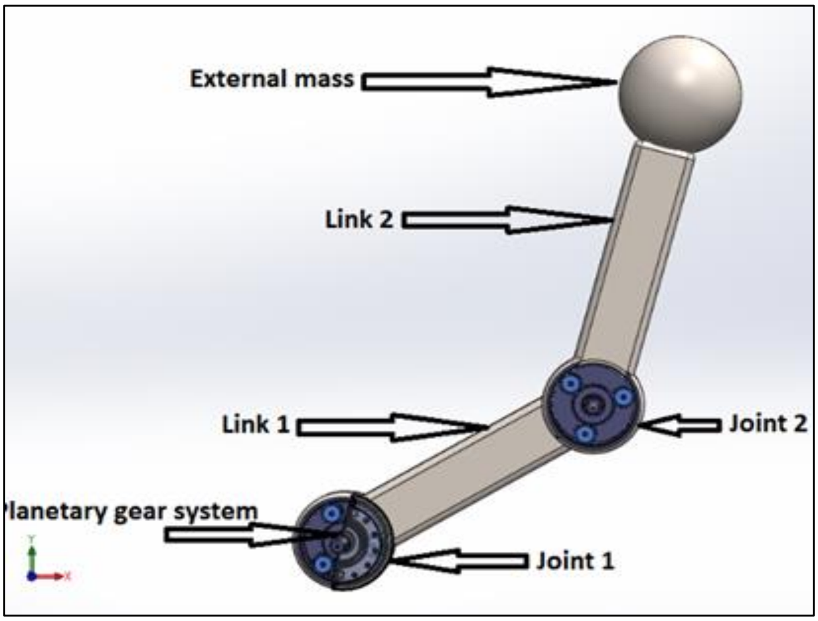

Figure 5 2-R robot actuated by planetary gear system modeled by using SolidWorks

Table 1 The mass properties of 2-R robot actuated by (n) equally spaced planet-gears by using SolidWorks

Parameter	**Value**	**Unit**
Mass of augmented link(1) actuated by 3 ESPG (m_i)	29	Kg
Mass of augmented link(1) actuated by 4 ESPG (m_i)	29.12	Kg
Mass of augmented link(1) actuated by 5 ESPG (m_i)	29.24	Kg
Mass of augmented link(1) actuated by 6 ESPG (m_i)	29.36	Kg
The inertia tensor of the augmented link(1) actuated by 3 ESPG ($\bar{I}^1_{1zz}$)	0.40	$Kg.m^2$
The inertia tensor of the augmented link(1) actuated by 4 ESPG ($\bar{I}^1_{1zz}$)	0.4028	$Kg.m^2$
The inertia tensor of the augmented link(1) actuated by 5 ESPG ($\bar{I}^1_{1zz}$)	0.4056	$Kg.m^2$
The inertia tensor of the augmented link(1) actuated by 6 ESPG ($\bar{I}^1_{1zz}$)	0.4083	$Kg.m^2$
The mass of the augmented link(2) (link+external mass)	29	Kg
The inertia tensor of the augmented link(2)(link+external mass) ($\bar{I}^{2o}_{2ozz}$)	0.43	$Kg.m^2$
The inertia tensor of the sun-gear ($I_{sg_{i+1(zz)}}$)	0.01574	$Kg.m^2$
The inertia tensor of the planet-gear ($I_{pg_{i+1(zz)}}$)	0.00314	$Kg.m^2$
The gear reduction ratios between the sun-gear and ring-gear ($G_{sgr,i+1} = G_{sgr,i}$)	2	
The gear reduction ratios between the planet-gear and ring-gear($G_{pgr,i+1} = G_{pgr,i}$)	4	

3.1. Kinetic energy simulation

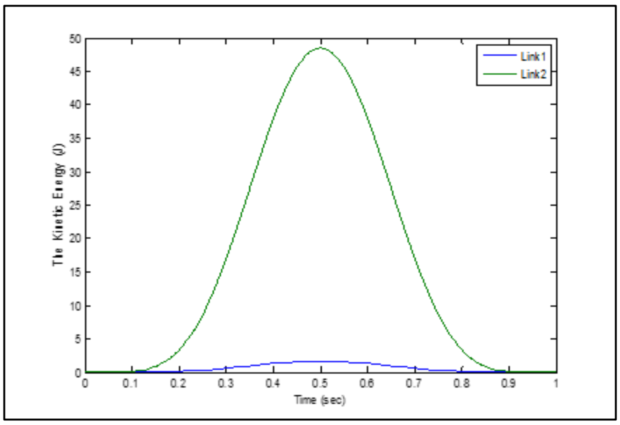

Figure 6 The kinetic energy of robot actuated by 3 ESPG

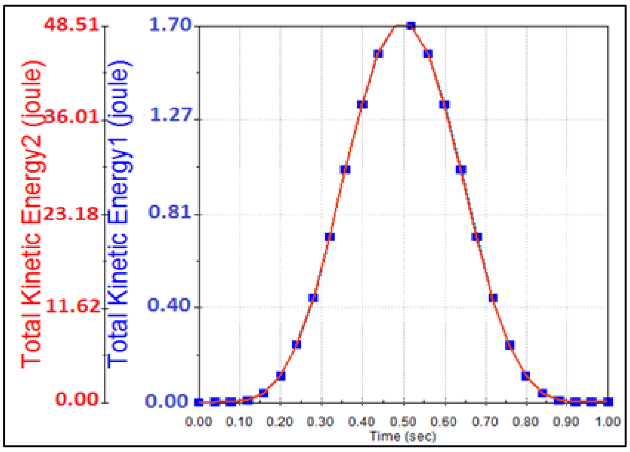

Figure 7 The kinetic energy of robot actuated by 3 ESPG by using SolidWorks

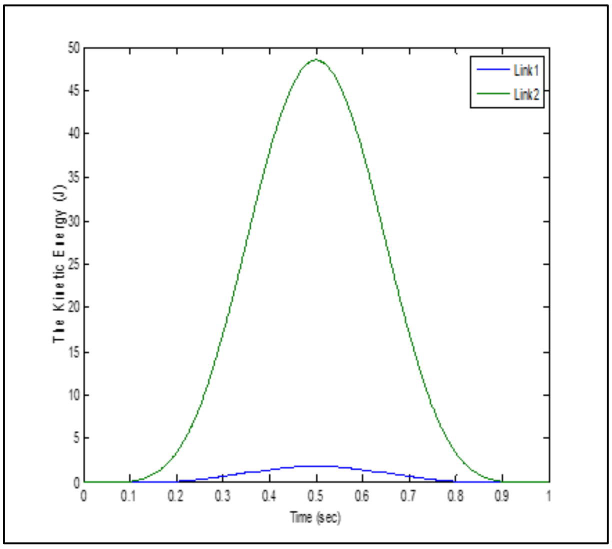

Figure 8 The kinetic energy of robot actuated by 4 ESPG

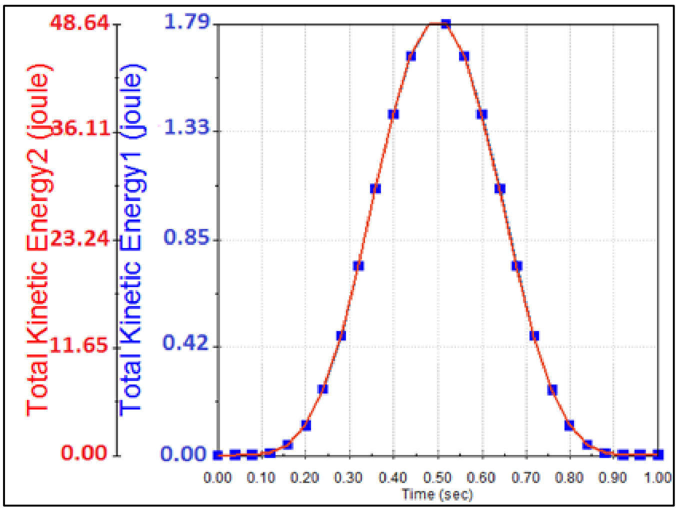

Figure 9 The kinetic energy of robot actuated by 4 ESPG by using SolidWorks

3.2. Joint torques simulation

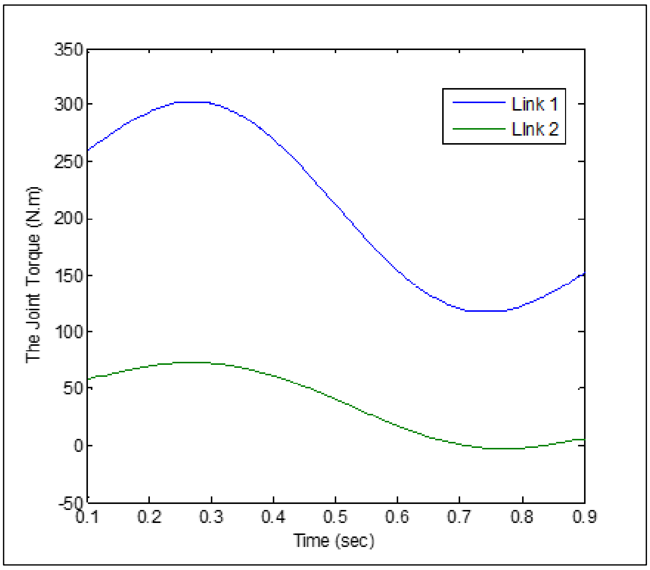

Figure 10 The variation of joint torques of links of robot actuated by 3 ESPG

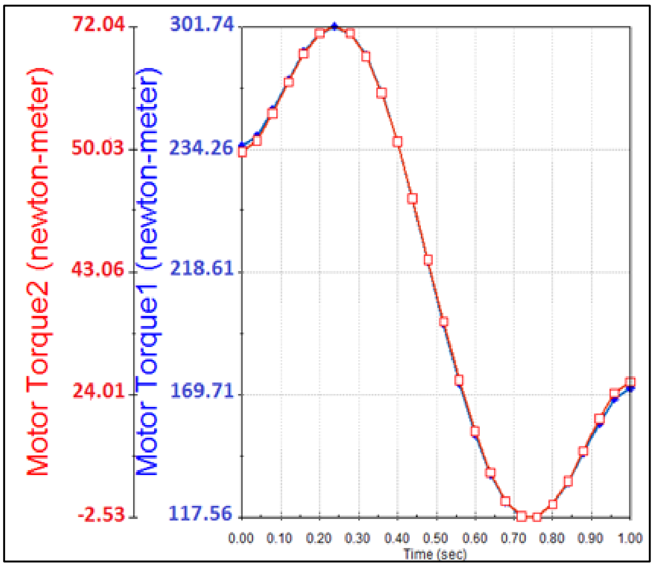

Figure 11 The variation of joint torques of links of robot actuated by 3 ESPG by using SolidWorks

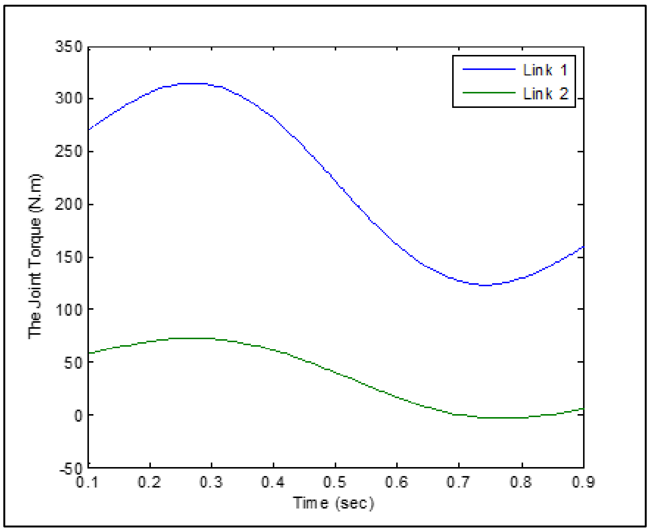

Figure 12 The variation of joint torques of links of robot actuated by 4 ESPG

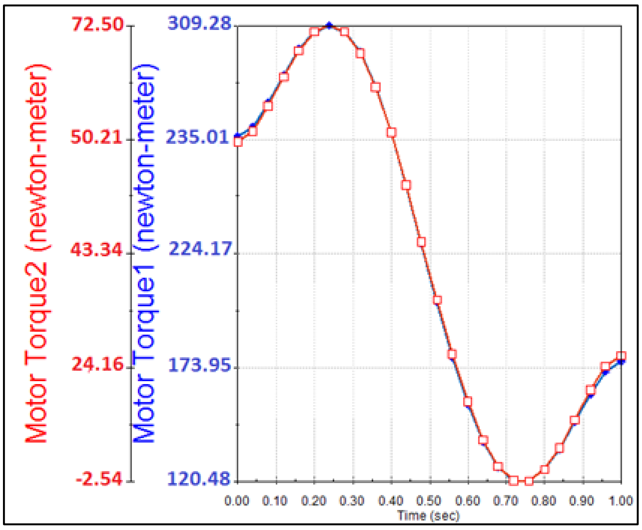

Figure 13 The variation of joint torques of links of robot actuated by 4 ESPG by using SolidWorks

4. Discussion

The dynamics modeling of robot actuated by (n) equally spaced planet-gears show that the kinetic energy of the augmented link shown in Eq. 57 and Eq. 63 (link+ sun-gear+planet-gears, link+external mass) is linear with respect to the dynamic parameters, namely, the mass, the three components of the first moment of inertia shown in Eq. 60 and Eq. 66, the six components of the inertia tensor shown in Eq. 62 and Eq. 68, and the moment of inertia of the sun-gear , planet-gears and external mass. The inertial effects of fast spinning sun-gear and planet-gears have a relevant influence on the dynamic behavior of manipulators. The effective sun-gear and planet-gears inertias are indeed multiplied by the square of gear ratios and the coupling torques and forces arise from the interaction with the link motion as shown in Eq. 57 and Eq. 89.

We can notice from the results obtained of the dynamic simulation of a 2-R robot shown in section. 3 that the results obtained, whether using simulation results or SolidWorks (2020) are the same, this similarity of results confirms the reliability and correctness of the dynamic model studied. [35-40]

Nomenclature

- C_i: Center of mass of augmented link.
- l_{i}: Center of mass of link (i).
- $\bar{I}_i$: Inertia tensor of augmented link (link+sun-gear+ planet-gears).
- $\bar{I}_{i0}$: Inertia tensor of augmented link (link+external mass).
- $I_{sg_{i+1}}$: Moment of inertia of sun-gear about its center of mass.
- $I_{pg_{i+1}}$: Moment of inertia of planet-gear about its center of mass.
- I_{e_m}: Moment of inertia of external mass about its center of mass.
- m_{e_m}: Mass of external mass.
- m_{l_i}: Mass of link (i).
- I_{l_i}: Moment of inertia of link (i).
- $m_{sg_{i+1}}$: Mass of sun-gear.
- $m_{pg_{i+1}}$: Mass of planet-gears.
- m_{i}: Mass of augmented link (link+sun-gear+planet-gears).
- m_{i0}: Mass of augmented link (link+external mass).
- r_{i-1,C_i}*:* Vector from origin of frame $(i-1)$ to center of mass C_i.
- r_{i,C_i}: Vector from origin of frame (i) to center of mass C_i.
- $r_{i-1,i}$*:* Vector from origin of frame $(i-1)$ to the frame (i).
- r_{i-1,l_i}*:* The vector from origin of frame $(i-1)$ to center of mass of link (i).
- r_{C_i,l_i}: The vector from the center of mass C_i to center of mass of link (i).
- $r_{C_i,sg_{i+1}}$: The vector from the center of mass C_i to the joint axis where the sun-gear is located.
- $r_{C_i,pg_{i+1}}$: The vector from the center of mass C_i to the center of mass of the planet-gear.
- r_{C_i,e_m}: The vector from the center of mass C_i to center of mass of external mass.
- $\dot{P}_{C_i}$: Linear velocity of center of mass C_i.
- $\ddot{P}_{C_i}$: Linear acceleration of center of mass $\mathrm{C_i}$.
- $\dot{P}_i$: Linear velocity of origin of frame (i).
- $\ddot{P}_i$: Linear acceleration of origin of frame (i).
- ω_i: Angular velocity of link (i).
- $\dot{\omega}_i$: Angular acceleration of link (i).
- ω_{sg_i}: Angular velocity of sun-gear.
- ω_{pg_i}: Angular velocity of planet-gear.
- $\dot{\omega}_{sg_i}$: Angular acceleration of sun-gear.
- $\dot{\omega}_{pg_i}$: Angular acceleration of planet-gear.
- g_0: Gravity accelaration.
- f_i: Force exterted by link $(i-1)$ on link (i).
- $-f_{i+1}$: Force exterted by link $(i+1)$ on link (i).
- μ_i: Moment exterted by link $(i-1)$ on link (i) with respect to origin of frame $(i-1)$.
- μ_{i+1}: Moment exerted by link $(i+1)$ on link (i) with respect to origin of frame (i).
- P_{C_i}: Vector from base coordinate system $(\mathrm{x_0,y_0,z_0})$ to position of center of mass of augmented link (i).
- P_{l_i}: Vector from base coordinate system $(\mathrm{x_0,y_0,z_0})$ to center of mass of link (i).
- $\dot{p}_{l_i}$: Linear velocity from base coordinate system $(\mathrm{x_0,y_0,z_0})$ to center of mass of link (i).
- $P_{sg_{i+1}}$: Vector from base coordinate system $(\mathrm{x_0,y_0,z_0})$ to the joint axis where the sun-gear is located.
- $P_{pg_{i+1}}$: Vector from base coordinate system $(\mathrm{x_0,y_0,z_0})$ to the center of mass of planet-gear.
- $\dot{p}_{sg_{i+1}}$: Linear velocity from base coordinate system $(\mathrm{x_0,y_0,z_0})$ to the joint axis where the sun-gear is located.
- $\dot{p}_{pg_{i+1}}$: Linear velocity from base coordinate system $(\mathrm{x_0,y_0,z_0})$ to the center of mass of planet-gear.
- P_{e_m}: Vector from base coordinate system $(\mathrm{x_0,y_0,z_0})$ to center of mass of external mass.
- e_m: The center of mass of external mass.
- r : The radius of external mass.
- r_{e_m,C_i}: The vector from the center of mass of external mass to center of mass of augmented link (i).
- S : Skew symetric matrix.
- R_i: The rotation matrix from Frame (i) to base frame $(\mathrm{x_0,y_0,z_0})$.
- $G_{sgr_{i+1}}$:The gear reduction ratios between sun-gear and ring-gear.

- $G_{pgr_{i+1}}$:The gear reduction ratios between planet-gear and ring-gear.
- Z_{rg}: The number of ring gear teeth.
- Z_{sg}: The number of sun-gear teeth.
- Z_{pg}: The number of planet-gear teeth.
- T_{l_i}: The kinetic energy of link (i).
- $T_{sg_{i+1}}$: The kinetic energy of the sun-gear.
- $T_{pg_{i+1}}$: The kinetic energy of the planet-gear.
- T_i : The Kinetic energy of the augmented link.

Abbreviation

- ESPG: Equally spaced planet-gears.

5. Conclusion

The dynamics modeling of robot holding an external mass and actuated by (n) equally spaced planet-gears are studied. The verification of the dynamic modeling of 2-R robot actuated by (n) equally spaced planet-gears by using the software SolidWorks (2020) permitted us to qualitatively develop and highlight the relevance of the dynamic model studied. These results are verified for 3-R planar robot and 3-R PUMA robot, and the dynamic modeling shown in section (2) are also verified by using Lagrange formulation.

References

[1] Prof. Dr. Giancarlo Genta, Actuators and Sensors Introduction to the Mechanics of Space Robots. In : Space Technology Library Volume26 Springer, pp 427-482 (2012

[2] Oriol Gomis-Bellmunt, Lucio Flavio Campanile,: Actuator Principles and Classification Design Rules for Actuators in Active Mechanical Systems. In : Springer, pp 3-28 (2010))

[3] Gomis-BellmuntO, Design,modeling, identification and control of mechatronic systems. In : PhD thesis, Technical University of Catalonia (2007)

[4] Gomis-Bellmunt O, Galceran-Arellano S, Sudri`a-Andreu A, Montesinos-Miracle D, : Linear electromagnetic actuator modeling for optimization of mechatronic and adaptronic systems. In : Campanile LF (2007)

[5] Reza,N,Jazar , : Theory of Applied Robotics Kinematics, Dynamics, and Control. In : (2nd Edition) Springer (2010)

[6] Bruno Siciliano, Lorenzo Sciavicco Luigi Villani, Giuseppe Oriolo, : Robotics Modelling, Planning and Control. In : Springer (2009)

[7] M.M. Bridges D.M. Dawson, : Redesign of robust controllers for rigid-link flexible-joint robotic manipulators actuated with harmonic drive gearing. In: IEE Proceedings Control Theory and Applocations, Vol. 142, Issue. 5. September (1995)

[8] Bruno Siciliano, Lorenzo Sciavicco Luigi Villani, : Lagrange and Newton-Euler dynamic modeling of a gear-driven robot manipulator with inclusion of motor inertia effects. In : Advanced Robotics (1995)

[9] Gyula Mester, Szilvester Pletl, Gizella Pajor, Zolth Jeges, : FLEXIBLE PLANETARY GEAR DRIVES IN ROBOTICS . In : IEEE International Conference on Industrial Electronics, Control, Instrumentation, and Automation, Power Electronics and Motion Control (1992)

[10] Koichi Koganezawa and Akira Ito, : Artificial Hand Based on the Planetary Gear System-Realization of Daily Utility Motion of a Hand with Minimum Actuators. In : IEEE International Conference on Mechatronics and Automation ICMA (2013)

[11] Hoyul Lee and Youngjin Choi, : A New Actuator System Using Dual-Motors and a Planetary Gear. In : IEEE/ASMETRANSACTIONS MECHATRONICS, VOL. 17, NO. 1, FEBRUARY (2012)

[12] Yan-feng CHEN, Xin-yue WU and Jing JIN, : Influence of the Sun Gear on the Dynamical Behavior of Planetary Gear Train . In : IEEE International Conference on Intelligent Human-Machine Systems and Cybernetics (2009)

[13] Murphy, S.H. ; Wen, J.T. ; Saridis, G.N, : Recursive calculation of geared robot manipulator dynamics. In : IEEE Proceedings International Conference on Robotics and Automation (1990)

[14] Paul, R. P, : Mathematics, Programming, and Control. In : Robot Manipulators The MIT Press, Cambridge, Mass (1981)

[15] Asada, H. and J.-J. E. Slotine, : Robot Analysis and Control. In : John Wiley and Sons (1986)

[16] Spong, M. W, : Modeling and Control of Elastic Joint Robots. In : Journal of Dynamic Systems, Measurement, and Control, December, Vol. 109, pp 310-319 (1987)

[17] A., P. Lugner, K. Desoyer, : Equations of Motion of Manipulators Including Dynamic Effects of Active Elements. In : IFAC Symposium on Robot Control (SYROC0'85), pages 425-430, (1985)

[18] Chen, J, : The Effects of Gear Reduction on Robot Dynamics. In : Proceedings of the NASA Conference on Space Telerobotics. JPL, Jan 31 - Feb 2 (1989)

[19] Walker, M. W. and D. E. Orin, : Efficient Dynamic Computer Simulation of Robotic Mechanisms. In : Journal of Dynamic Systems, Measurement, and Control, Vol. 104, pp 205-211 September (1982)

[20] H.Randy shih, : Introduction to finite element analysis using Solidworks simulation 2012. In : oregon institute of technology (2012)

[21] J. Ed Akin, : Finite Element Analysis Concepts via SolidWorks . In : Rice University, Houston, Texas (2009)

[22] Michael Brand, : FEM-Praxis mit SolidWorks Simulation durch Kontrollrechnung und Messung verifizieren. In : springer (2013)

[23] John J Craig, : Introduction to robotics Mechanics and control . In : Third edition 2005.

[24] R. Mathis and Y. Rémond, : Une théorie unifiée des trains épicycloïdaux. In : Machine, Mécanismes, Robotiques, C.R. Acad. Sci. Paris, t.327, Série II.b, pp. 1115–1121, (1999)

[25] Yuki Kubo, Satoshi Ogasawara, and Masatsugu Takemoto, : Equivalency of Series Parallel Hybrid System Using Planetary Gear and Hybrid System Using EVT . In : IEEE International Conference on Power Electronics - ECCE Asia The Shilla Jeju, Korea May 30-June 3 (2011)

[26] Roland Mathis, Yves Remond,: Kinematic and dynamic simulation of epicyclic gear trains. In : Mechanism and Machine Theory February (2009).

[27] Corke Peter, : A robotics toolbox for MATLAB. In : IEEE Robotics & Automation Magazine (1996)

[28] M.RENAUD, Quasi-minimal computation of the dynamic model of robot manipulator utilizing the Newton-Euler formalism and notion of augmented body. In: IEEE International Conference on Robotics and Automation (1987)

[29] Sisir K. Padhy, : On the dynamics of SCARA robot. In : Robotics and Autonomous 71-78 Systems 10 (1992)

[30] A.Gasparetto, A.Lanzutti, R.Vidoni, V.Zanotto, : Experimental validation and comparative analysis of optimal time-jerk algorithms for trajectory planning. Robotics and Computer-Integrated Manufacturing 164–181 (2012)

[31] P. J. Barre, R .Bearee, P .Borne, E .Dumetz, : Influence of a jerk controlled movement lawon the vibratory behaviour of high-dynamics systems. In : Journal of Intelligent and Robotic Systems ; 42(3) 275–93 (2005)

[32] Sonja Macfarlane and Elizabeth A. Croft, Member, : Jerk- Bounded Manipulator Trajectory Planning Design for Real-Time Applications. In : IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, VOL. 19, NO. 1, FEBRUARY (2003)

[33] A. Piazzi and A. Visioli, : Global minimum-jerk trajectory planning of robot manipulators. In : IEEE Trans. Ind. Electron., vol. 47, pp. 140–149, Feb. (2000)

[34] Brahim FERNINI, Mustapha TEMMAR, : An algorithm for serial planar manipulators that have an architecture R// (n) R by using Solidworks (2013) and Matlab/Simulink (2012). In : IEEE International Confrence on Mechatronics and Automation ICMA (2013)

[35] Mahmoud GOUASMI, Mohammed OUALI, Brahim FERNINI, M'hamed MEGHATRIA, : Kinematic modeling and simulation of a 2-R robot using Solidworks and verification by Matlab/Simulink. In : International journal of advanced robotic systems (2012)

[36] Brahim Fernini. Mathematical modelling and simulation investigation of the dynamic behaviour of a compliant 2-R robot by using NE method Via Matlab/Simulink (2022): International Journal of Frontiers in Engineering and Technology Research

[37] B.Fernini, M.Temmar and MM Noor. Toward a dynamic analysis of bipedal robots inspired by human leg muscles: Journal of Mechanical Engineering and Sciences (2018)

[38] B Fernini, M Temmar, Y Kai, MM Noor. Verification of the dynamic modeling of 2-R robot actuated by (N) equally spaced planet-gears by using SolidWorks and MATLAB/SIMULINK: Mechanics and Mechanical Engineering (2018)

[39] B Fernini, M Temmar. The effect of mono and biarticular muscles on the dynamic of walking bipedal robot: Intelligent Systems Conference (IntelliSys) (2017)

[40] B Fernini. Dynamic Behavior of a SCARA Robot by using NE Method for a Straight Line and Simulation of Motion by using Solidworks and Verification by Matlab/Simulink: International Journal of Robotics and Automation (lJRA) (2014)