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Further study of stress-strain deformation of some structural reinforcement steel rods with different diameters from a mini mill in Nigeria using theoretical and regression analysis

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Abstract

Further study of stress-strain deformation of some structural reinforcement steel rods with different diameters from a mini mill in Nigeria using theoretical and regression analysis has been undertaken. The work utilized result test of mechanical testing carried out on different sizes of reinforcement steel bars for concrete reinforcement ranging from 10 mm to 28mm in diameter. The deformation pattern of the work was highlighted using stress-strain graph and subjected to theoretical analysis, where the result showed that it is a ductile material with all the deformation regions associated with a ductile material. It equally has an elongation % of 9.36 at the point of failure during the test. Values of % elongation, ultimate tensile strength generated with different sizes of reinforcement steel bar were subjected to different regression models to establish their relationship, and to also find out which model best fit the relationship between the diameter variation of the steel rod and the % elongation, and the diameter variation of the steel rod and the ultimate tensile strength. The result showed that the relationship was linear, and linear regression model was better than hyperbolic curve model, and exponential function model. Therefore linear regression model was used to develop prediction model equations to estimate the values of % elongation and ultimate tensile strength. These models were evaluated using coefficient of determination r^2, standard error of regression, confidence limits, standard errors of the intercept (a) and the gradient (b), confidence interval for intercept and gradient, and finally significance test was carried out on the intercept and the gradient. The standard error of regression for model equation Y_1 was very small; 0.37, and that of model equation Y_2 was 60.18. The coefficient of determination r^2 was 21% for model equation Y_1 and 1.49% for model equation Y_2. The results also show that the general confidence interval has a narrower range than the individual confidence interval. The rank correlation coefficient has indicated that the association of the diameter variation of the steel rod was in perfect negative to the % elongation at failure and ultimate tensile strength. In conclusion this work has further thrown light to the stress-strain deformation of different diameters of structural reinforcement steel rods.

Keywords: Further Study; Mini Mills; Diameter; Steel bar; Stress-Strain; Deformation; Stiffness

1. Introduction

The failure of engineering materials, is almost always an undesirable event for several reasons, these include human lives that are put in jeopardy, economic losses, and the interference with the availability of products and services [1] ~~(Khanna, 2009)~~. A case in hand is the collapse of a 21 storey building in Lagos- Nigeria and that of a multi-storey building in Port-Harcourt-Nigeria, both of which claimed several lives, including millions of money invested in the two commercial building projects [2]. Even though, the causes of failure and the behavior of materials may be known, prevention of failures is difficult to guarantee. The usual causes are improper materials selection and processing and

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inadequate design of the component or its misuse. It is the responsibility of the design engineer to anticipate and plan for possible failure and, in the event that failure does occur, to assess its cause and then take appropriate preventive measures against future incidents ([3];[4];[1]).

Even though, the number of failures of a particular component may be small, they are important because they may affect the manufacturer's reputation for reliability. In some cases, particularly when the failure results in personal injury or death, it will lead to expensive lawsuits. With incessant collapse of building structures in Nigeria, studies have clearly shown that it is irresponsible to sorely lay the blame on reinforcement steel rods used in building structures ([5];[4];[6];[2]). Therefore, from the foregoing, once a component starts failing, its analysis is very essential. Laboratory and field testing permit the evaluation of the effects of material, design, and fabrication variables on performance of the part under controlled conditions. Failure analysis, on the other hand, is concerned with parts returned from service and thus gives results of actual operating conditions. This work however, focuses on the laboratory and field testing of structural reinforcement steel rebars from a mini-mill in Nigeria. The recent happenings of building structure failure has called for the need to further assess the stress-strain deformation of some of these steel rebars to establish their characteristics.

The strength of these rebars is important but it cannot be considered entirely at the expense of ductility. This characteristic introduces safety into reinforced concrete structures, so to say, the plastic deformation of the ductile steel reinforcement rod allows for strain hardening of the reinforcement rod, which further strengthens the concrete structure. The ductile failure of the steel rod which allows for necking before failure also gives rise to sagging of the structure before failure, this allows for time for workers or occupants to escape the building before failure. This mode of failure is completely different from brittle failure which is sudden and catastrophic without warning. Standard test specimens are sometimes used in the laboratory to draw inferences on the materials, failure analysis prefer to test the specimens as they are received from the service condition. Are there really major differences in results? Analyst have argued that there are differences ([7];[8]; [1]; [9).

Strain hardening or work hardening is a phenomenon which results in an increase in hardness and strength of a metal (specimen) subjected to plastic deformation (cold working) at temperatures lower than the recrystallization range. An important characteristic of plastic deformation of metals is that the shear stress required to produce slip continuously increases with shear strain. This increase in the stress required to cause slip because of previous plastic deformation or the increase of strength of material due to mechanical working is known as strain hardening or work hardening [1] ~~(Khanna, 2009)~~. The objective of this research work is to further study stress-strain deformation of some structural reinforcement steel rods with different diameters from a particular mini mill in Nigeria, using regression analysis and determining some inferential effects from the size variation on some characteristics of the steel rods.

2. Material and methods

2.1. Materials and Equipment

The materials used for the research work were ribbed reinforcement steel bars collected from a particular mini mill at Lagos-Nigeria. The equipment utilized in the quality analysis of the samples included; files, hack saw, lathe machine, Vernier calipers, protractor, universal strength testing machine.

2.2. Sample Collection

To actualize this project; different sample sizes of reinforcement steel bars were collected from a particular mini mill in Lagos-Nigeria.

2.3. Tensile Test

The only mechanical test carried out on the samples was tensile test. This was informed by the fact that in service reinforcement rods embedded in the concrete structure handle the tensile component of the stress on the structure. The compressive component of the stress on reinforced structures are mainly handled by the concrete cast. The six (6) samples were sent to Mudiame International Limited, Port-Harcourt-Nigeria for the tensile tests. % elongation which is a measure of ductility of the specimen was determined by obtaining the length of a test piece, putting the pieces together after the failure, and measuring the final length of the test piece. The % elongation was then calculated. All the samples were tested according to reference code / standard: BS 4449:2015+A3:2016 [9]. The results were plotted on graph and tests results were tabulated.

3. Results

The results of the tests are shown in Table 1 and the plot of the graph of specimen 1 shown in Fig. 1.

Table 1 Tensile Test for Six (6) Sizes of Reinforcement Steel Rods from a Mini Mill

Specimen	Diameter (mm)	Cross sectional area (mm^2)	Ultimate tensile strength (MPa)	% Elongation at failure
1	10	78.54	714	9.36
2	12	113.10	584	9.34
3	16	201.00	428	9.32
4	20	314.00	338	9.30
5	24	452.20	279	9.20
6	28	615.44	238	9.00

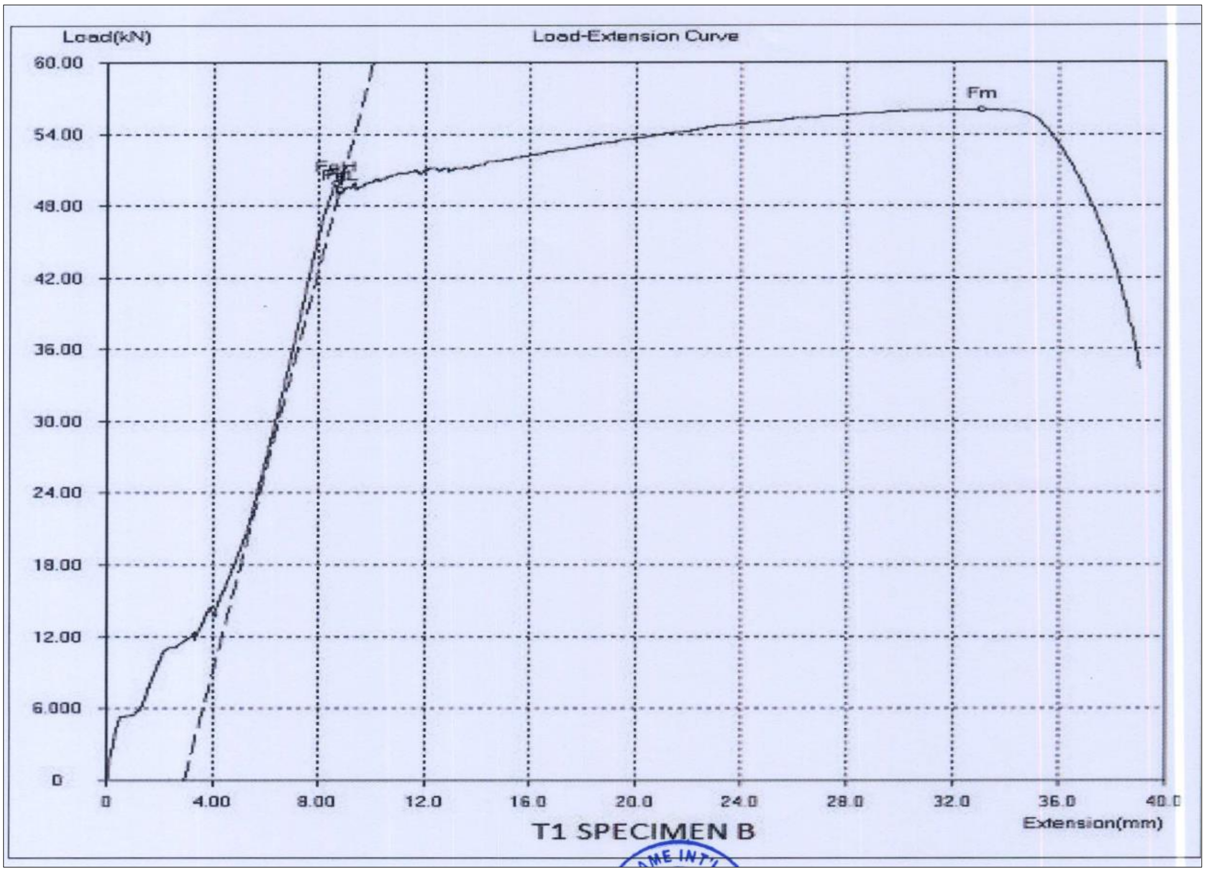

Figure 1 Load-Extension Curve for Specimen 1

4. Discussion

4.1. Theoretical Analysis

Fig 1 is the load –extension curve or stress-strain curve for specimen 1. It illustrates the deformation pattern of the 10 mm diameter reinforcement steel rod. The bar under loading exhibited the elastic region, the yield point; both the upper and lower yield point, it shows the region of plastic deformation (this is where permanent deformation of the metal sets in. The plastic region showed the point of ultimate tensile strength which correspond to the maximum load or stress, the plastic region also shows where necking started and final failure occurred. The curve bent and tipped downward at these last points. The elastic region is not straight from the origin of the curve this could be attributed to the gripping problem from the machine as soon as the clamping was stabilized the elastic region became straight up to the yield point. The deformation pattern in fig. 1 is referred to as ductile deformation or failure. It is an indication that the specimen is a ductile material ([7]; [10]; [11]). This type of deformation pattern is very important in reinforcement rebars because it prevents, sudden failure in concrete reinforced structures. The result of the test shows that the specimen has a % elongation of 9.36 and an ultimate tensile strength of 714 MPa. The strength is quite high, but the % elongation is low, but reasonable for that kind of strength. For plain carbon steel when the carbon content increases the strength increases, but the ductility decreases. The % elongation is 9.36 times more than that of a brittle material which is just 1% ([11]). Khanna (2009) [1] said that when a concrete member is bent, failure occurs on the tension side of the member, resulting in cracks in the concrete mass. To overcome this weakness reinforced concrete has been designed in which steel in the form of rods, wires, bars, or fabric is embedded in the fresh concrete. This minimizes the development of tensile stresses in concrete and produces material of much greater strength in compression, shear, and tension. In reinforced concrete, steel bars carry the tensile load/stresses (Khanna, 2009) [1].

According to Higgins (1985) [7], when corresponding values of stress and strain derived during a tensile test is plotted graphically as in Fig.1. It is found that each type of material is represented by a characteristic curve. Materials of negligible ductility, such as fully hardened steels, cast iron and concrete, undergo little or no plastic deformation before fracture. That is, there is no yield point and only elastic extension occurs. A ductile material, on the other hand, exhibits an elastic limit (or limit of proportionality) beyond which plastic deformation occurs. The maximum stress which a material can withstand before plastic flow sets in is known as its yield strength. In softer ferrous materials (wrought-iron and low-carbon steels) and some plastic materials, the onset of plastic flow is marked by a very definite yield point and it is therefore a simple matter to calculate the yield stress. In other materials, comprising practically all ductile metals and alloys and most plastic materials, the elastic limit is not well defined. In most respects the yield stress of a material is of greater importance to the design engineer than is the maximum strength attained during plastic flow.

Consequently a substitute value for yield strength is derived for those materials which show no obvious yield point. This is known as the proof stress and is that stress which will produce a permanent (plastic) extension of 0.1% in the gauge length of the test piece. Materials which have received some treatment such as work-hardening or in the case of some alloys, suitable heat-treatment, are generally stronger but less ductile than those in the fully soft condition. Higgins (1985) [7] explanation of the stress-strain diagram for ductile material agrees with the stress-strain diagram of Fig.1.

4.2. Statistical Analysis

The results of Table 1 are used in the statistical analysis. Efforts have been made in the work to construct models that would best explain the relationship between the dependent variable Y_1 (% elongation) and the independent variable X (different steel reinforcing sizes). And also, the relationship between dependent variable Y_2 (ultimate tensile strength) and the independent variable X (different steel reinforcing sizes). Several regression models were tested to construct the line of best fit for the result, however, most of the models were not suitable in explaining the relationship between the dependent and the independent variables. Details is as presented below:

4.3. The Hyperbolic curve models

$$Y = a + \frac{b}{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The values of a and b are calculated by reference to amended formulae:

$$b = \frac{n\Sigma\left(\frac{1}{x}\right)Y - \Sigma\left(\frac{1}{x}\right)Y}{n\Sigma\left(\frac{1}{x}\right)^2 - \left(\Sigma\frac{1}{x}\right)^2} \quad a = \frac{\Sigma Y}{n} - \frac{b\Sigma\left(\frac{1}{x}\right)}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Using the values in Table 1 and arranging a second table in accordance with equation 2, equation 2 can be solved and the values for a and b substituted into equation 1to construct the hyperbolic curve models:

$$Y = a + \frac{b}{x} \text{ Eq (1) now becomes } Y_1 = -256.1 + \frac{4291.29}{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$Y_2 = -11904.96 + \frac{199490}{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Prediction values derived using model eqns 3 and 4 did not agree or give close values to experimental results in Table 1. Relationship between Y_1 and x and Y_2 and x cannot be hyperbolic curve model and therefore line of best fit cannot be plotted using eqns. 3 and 4 ([12]; [13]).

4.4. Exponential Function Model

$$Y = ab^x \text{ .... } (5)$$

$$logY = loga + xlogb \text{ or } logY = A + Bx \text{ .... } (6)$$

where, A = log a and B = log b.

Solving eqn 6 for the values of a and b and substituting in eqn 5 using values from Table 1 we can construct exponential model eqn:

$$Y = ab^x = Y = 767200(0.1)^x \text{ .... } (7)$$

Prediction values derived using model eqn 7 which is supposed to describe the relationship between reinforcement steel rod size and % elongation did not give close values to the experimental values in Table 1. Therefore the relationship between the % elongation and the diameter of the reinforcement steel rods cannot be predicted using exponential function model. The relationship between Y and x is not an exponential function.

4.5. Linear Regression Model

In the general form of the equation for a straight line

$$Y = a + bx \text{ .... } (8)$$

$$a = \frac{\Sigma Y}{n} - \frac{b\Sigma x}{n} \quad b = \frac{n\Sigma xY - \Sigma x\Sigma Y}{n\Sigma x^2 - (\Sigma x)^2} \text{ .... } (9)$$

Where, n = number of pairs of figures, the slope of the line b is sometimes called the regression coefficient.

From Table 1 two tables are constructed to find the values of a and b using equation 9 to determine model equations for Y_1 and Y_2.

Table 2 Regression analysis of x on Y_1

X	Y_1	X^2	Y_1^2	XY	Model equation values of Y_1
10	9.36	100	87.61	93.6	9.320
12	9.34	144	87.24	112.08	9.302
16	9.32	256	86.86	149.12	9.268
20	9.30	400	86.49	188.00	9.230
24	9.20	576	84.64	220.80	9.194
28	9.00	784	81.00	252.00	9.158
∑110	55.52	2260	513.84	1015.80	

Using eqn. 9 the value of a = 9.41 and the value of b = - 0.009

Substituting the values of a and b into equation 8 the model equation for the regression of x on $Y_{1\text{ becomes}}$:

$$Y_{1= 9.41-0.009X} \text{ .... } (10)$$

Equivalent values of Y_1 using the model equation (10) are shown in Table 2. This same equation can be used to predict the % elongation of the steel reinforcement rod (Y_1) when the diameter of the steel rod (X) is 30mm. Y_1 = 9.41- 0.009(30) = 9.14%

Table 3 Regression Analysis of x on Y_2

X	**Y_2**	**X^2**	**Y_2^2**	**XY**	**Model equation values of Y_2**
10	714	100	509798	7140	642.55
12	584	144	341058	7008	591.56
16	428	256	183184	6848	489.64
20	338	400	114244	6760	387.72
24	279	576	77841	6698	285.80
28	238	784	56644	6664	183.88
∑110	2581	2260	1282769	41118	

Using eqn. 9 the value of a = 897.315 and the value of b = - 25.48

Substituting the values of a and b into eqn.8, the model equation for the regression of x on Y_2 becomes:

$$Y_2 = 897.315 - 25.48X \quad \text{……………………………..} \ (11)$$

Equivalent values of Y_2 using model equation (11) are shown in Table 3. This same equation can be used to predict the ultimate tensile strength of the steel rod (Y_2) when the diameter of the steel rod (X) is 30mm. $Y_2 = 897.315 - 25.48(30) = 132.915$ The variation between the experimental results and the model equation results seem to be quite wide, this obviously is from the trend of the experimental results ([13]). However, the model eqn (11) provides a moderate line of best fit, all things being equal ([12]).

4.6. Coefficient of Determination Validating Linear Regression Model Equation

To find out how good the line of best fit really is, a measure called the coefficient of determination is calculated. This measure denoted by r^2 (because it is the square of the correlation coefficient, r) calculates what proportion of the variation in the actual values may be predicted by changes in the values of X.

Thus, r^2 is the ratio

$$\frac{Explained\ Variation}{Total\ Variation} = \frac{\Sigma(YE - \bar{y})^2}{\sum(Y - \bar{y})^2} \quad \text{……………………………..} \ (12)$$

where, YE = Estimate of Y given by the regression equation for each value of x

$\bar{y}$= mean of actual values of Y

Y_1 = Individual actual values of Y

$$Y_1 = 9.41 - 0.009X$$

$$\bar{y} = \frac{55.52}{6} = 9.25$$

Table 4 Calculation of r^2

X	Y_1	YE	YE- $\bar{y}$	(YE- $\bar{y}$)^2	Y_1- $\bar{y}$	(Y_1-$\bar{y}$)^2
10	9.36	9.32	0.07	0.0049	0.11	0.012
12	9.34	9.30	0.05	0.0025	0.09	0.008
16	9.32	9.27	0.02	0.0004	0.07	0.005
20	9.30	9.23	-0.02	0.0004	0.05	0.003
24	9.20	9.19	-0.06	0.0036	-0.05	0.003
28	9.00	9.16	-0.09	0.0081	-0.25	0.063
∑	55.52			0.0199		0.094

$$r^2 = \frac{Explained\ variation}{Total\ variation} = \frac{\sum(YE - \bar{y})^2}{\sum(Y - \bar{y})^2} = \frac{0.0199}{0.094} = 0.21$$

$$\therefore 100r^2 = 21\%$$

The result shows that 21% of the variation in the elongation % of the reinforcement steel bar may be predicted by change in the actual value of the reinforcement steel bar diameter X. Factors other than changes in the diameter of the steel bar account for 79% of the variation in % elongation Y_1 ([14]; [12]; [6]).

$\boldsymbol{r^2}$ for equation 11

$$Y_2 = 897.315 - 25.48X \text{ ………………………….. (11)}$$

$$\bar{y} = \frac{2581}{6} = 430.17$$

$$r^2 = \frac{(n\sum XY - \sum X \sum Y)^2}{(n\sum X^2 - (\sum X)^2\,(n\sum Y^2) - (\sum Y)^2)} \text{ ………………………….. (13)}$$

From equation 13

$$r^2 = 0.0149 \text{ and } 100r^2 = 1.49\%$$

The result have shown that 1.49% of the variation in the ultimate tensile strength of the steel reinforcement may be predicted by change in the actual value of the reinforcement steel bar diameter X. Other factors other than changes in the diameter of the steel bar accounts for 98.51% of the variation in the ultimate tensile strength Y_2.

4.7. Standard Error of Regression for Equation 10

$$Y_1 = 9.41 - 0.009X$$

Standard error of regression (residual standard deviation) = Se = $\sqrt{\frac{\sum Y^2 - a\sum Y - b\sum XY}{n-2}}$ ………………………….. (14)

Putting into equation (14) the appropriate values of a, b, n, ∑Y, ∑XY, and $\sum Y^2$ we have;

$$Se = 0.37$$

Note: a , b are from equation 10 while the other terms are from previous calculations above. As previously observed above, the standard error of regression for model equation 10 is quite small; 0.37, the model equation therefore is good enough to predict the % elongation of different sizes of steel reinforcement rods with minimal errors. The equation is capable of giving approximate values of how ductile the steel is.

4.8. Standard Error of Regression Equation 11

$$Y_2 = 897.315 - 25.48X$$

From equation 14

$$Se = 60.18$$

Note: a and b are from equation 11 while the other terms are values from previous calculations above.

As previously observed above it can be seen that the standard error for equation 11 is very high, although it may be the best line of fit for the ultimate tensile strength, but where accuracy of prediction is required the model equation cannot be used to predict ultimate tensile strength of different sizes of steel reinforcement rods ([12]).

4.9. Setting Confidence Limits

For equation 10

9.41 – 0.009X

The confidence limits for the whole of the regression line are calculated by using a quantity known as the standard error of the average forecast which is given by

$$S_{ef} = S_e \sqrt{\frac{1}{n} + \frac{(x - \ddot{x})^2}{\sum X^2 - \frac{(\sum X)^2}{n}}} \text{ .... } (15)$$

$$\frac{1}{n} = \frac{1}{6} = 0.17 \text{ ; } \sum X^2 - \frac{(\sum X)^2}{n} = 2260 - \frac{12100}{6} = 2260 - 2016.67 = 243.33$$

$$\ddot{x} = \frac{110}{6} = 18.33$$

4.10. Constructing the confidence interval

The actual confidence interval is constructed in exactly the same way as that for a mean or a proportion. In this case the number of observations are 6, then the't' distribution is used with 6-2 = 4 degree of freedom. The interval is calculated by estimating the fitted values of Y for each value of X in the original data using the equation

$Y = a + bX$ The interval then takes the form

$$Y \pm S_{ef} X t \text{ .... } (16)$$

From equation 10

$$a = 9.41; b = -0.009, calculated\ above\ S_e = 0.37$$

Using equations 15 and 16 the confidence interval is constructed as shown in Table 5

Table 5 Confidence interval for equation 10

Confidence interval			
X	**Y**	**Lower limit**	**Upper limit**
10	9.32	8.63	10.01
12	9.302	8.71	9.896
16	9.268	8.82	9.718
20	9.23	8.79	9.668
24	9.194	8.63	9.759
28	9.158	8.39	9.923

4.11. Confidence interval for individual predictions of X

The amended formula calculates what is known as the standard error of the individual forecast and is shown below:

$$S_e(individual) = S_e\sqrt{1+\frac{1}{n}+\frac{(X-\ddot{x})^2}{\sum X^2-\frac{(\sum X)^2}{n}}} \quad \text{………………………………} (17)$$

Using equation 17 for X = 12

$S_e(individual) = 0.428$

Confidence interval = 9.302 $\pm$ 1.187

Lower limit = 8.115 while upper limit is 10.489

These values if contrasted with the values obtained for the general confidence intervals in Table 5, when X = 12 the range is 8.71 to 9.896, thus, it can be seen that when an individual prediction of values of Y is made the confidence intervals are much wider.

4.12. Confidence Intervals for Equation 11

897.315 – 25.48X

The confidence interval for equation 11 is as shown in Table 6

Table 6 Confidence interval for equation 11

Confidence interval			
X	**Y**	**Lower limit**	**Upper limit**
10	642.515	641.82	643.21
12	591.559	590.97	592.15
16	489.635	489.19	490.09
20	387.715	387.28	388.15
24	285.799	285.23	286.36
28	183.875	183.11	184.64

Individual confidence interval value calculation when X = 12

$$Y = 591.559 \pm 1.187$$

The lower limit is 590.37 while the upper limit is 592.75

When the individual confidence interval for X = 12 is contrasted with the general confidence interval in Table 6, it can be seen that an individual prediction of the value of Y has a wider confidence interval.

4.13. Standard Errors of the Intercept (a) and the gradient (b)

$$\text{The intercept } S_a = S_e \sqrt{\frac{\sum X^2}{n\sum X^2 - (\sum X)^2}} \text{ ……………………………. (18)}$$

$$\text{The gradient } S_b = \frac{S_e}{\sqrt{\sum X^2 - \frac{(\sum X)^2}{n}}} \text{ ……………………………. (19)}$$

Where Se is the standard error of regression

For equation 10

$$9.41 - 0.009X$$

Intercept $S_a\ from\ equation\ 18 = 0.46$

$$\text{The gradient } S_b\ from\ equation\ 19 = 0.024$$

The confidence intervals for α and β may be established as follows:

For intercept

$$\alpha = a\ \pm txS_a = 9.41 \pm 1.277 = \ upper\ and\ lower\ limits\ of\ 10.69\ and\ 8.13$$

For the gradient

$$\beta = b\ \pm txS_b = \ -0.009\ \pm 0.0666 = \ upper\ and\ lower\ limits\ of\ 0.0576\ and\ 0.0756$$

4.14. Test of Significance for α and β

For the intercept

$$H_o : \propto = 0$$

$$H_1 : \propto \neq 0$$

The test statistics is

$$t = \frac{a - \propto}{S_a} = \frac{9.41 - 0}{0.46} = 20.46$$

Since 20.46 is much greater than 2.776 (the value from t tables) Ho hypothesis can be rejected.

For the slope β

$$H_o: \beta = 0$$

$$H_1: \beta \neq 0$$

The test statistics or significance test for the slope is

$$t = \frac{b-\beta}{S_b} = \frac{-0.009-0}{0.024} = -0.375$$

Since -0.375 is much less than 2.776 (table value) we accept Ho hypothesis.

Equation 10 can be used as a basis of predicting the ductility of the reinforcement steel rods of various sizes.

For equation 11

897.315- 25.48X

For intercept

$$\alpha = a \pm txS_a = 897.315 \pm 1.277 = upper\ and\ lower\ limits\ of\ 898.59\ and\ 896.04$$

For the gradient

$$\beta = b \pm txS_b = -25.48 \pm 0.0666 = upper\ and\ lower\ limits\ of - 25.55\ and\ 25.41$$

Test of Significance

For the intercept

$$H_o : \propto = 0$$

$$H_1 : \propto \neq 0$$

$$t = \frac{a - \propto}{S_a} = \frac{897.315-0}{0.46} = 1950.69$$

Since the calculated value is far more than the table value of 2.776 we reject Ho.

For the gradient

$$H_o: \beta = 0$$

$$H_1 : \beta \neq 0$$

$$t = \frac{b-\beta}{S_b} = \frac{-25.48-0}{0.024} = -1061.67$$

Since -1061.67 is less than 2.776 (table value), Ho can be accepted. On the basis of this evidence the regression equation Y = 897.315 - 25.48X can be used as a basis of prediction for the ultimate tensile strength of the tested reinforcement steel rod.

4.15. Rank Correlation Coefficient (R)

This coefficient is also known as the Spearman rank correlation

$$R = 1 - \frac{6\sum d^2}{n(n^2-1)} \text{ ………………………….. (20)}$$

Where, n = number of tests values

Table 7 Calculating R for steel rod diameter versus % elongation

Ranking of steel diameter	Ranking of % elongation	d (difference between ranks)	d2
6	1	5	25
5	2	3	9
4	3	1	1
3	4	-1	1
2	5	-3	9
1	6	-5	25
			$\sum d^2 = 70$

Substituting the values in Table 7 into equation 20, R= -1.

As the Rank correlation coefficient is -1 we are able to say that there is a reasonable agreement between the size variation of steel reinforcement rod and % elongation of the steel, it is however, in the negative.

Calculating R for the values of Steel rod diameter versus ultimate tensile strength, the value of R is also in the negative. The association between the two variables is in the negative. If one variable is increasing the other will be decreasing, it could also mean no strong association because other factors may be directly responsible for the variation. However, considering the equations below:

$$\sigma = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{F}{\pi\left(\frac{d}{2}\right)^2} = \frac{4F}{\pi d^2} \quad \text{.................................} (21)$$

$$\sigma = E\varepsilon = \frac{4F}{\pi d^2} = E\varepsilon \quad \text{.................................} (22)$$

where, σ in this case is the ultimate tensile stress, F is the force, d is the diameter of the reinforcement steel rod, E is the Young's modulus of elasticity, ε is the strain and A is the cross-sectional area of the reinforcement steel rod ([7]; [12]; [1]; [5]).

From equations 21 and 22 it can be seen that the equations completely agrees with the result of the rank correlation coefficient that the diameter of the steel rod is in a negative association with the ultimate tensile strength, and % elongation of the steel rod at failure ([12]) The increase of the diameter will lead to the decrease of the two other variables.

The statistics has shown that just 21% in % elongation is due to size variation of the reinforcement steel rod. Other factors constitute 79% of the variation in the % elongation of the reinforcement steel rod; these include, microstructure, composition, and method of production ([14] [12] [8]; [4]). The statistics also show that size variation of the reinforcement steel rod has only 1.49% influence on the variation of ultimate tensile strength of the reinforcement steel rod. Other factors are responsible for the bulk of the variations. This statistics to some extent explains why standard test specimens are used in mechanical testing. Bulk of the factors determining the mechanical properties of the steel are inherent and does not depend much on the size of the steel, provided the standard specimen is prepared to specification and very smooth. The as -received testing of specimens is encouraged because the test is done in the condition which the specimen is in, without interference. Stiffness of the material is however, increased by size, but the mechanical properties like yield strength, ultimate tensile strength, and % elongation depend on composition, structure and method of manufacture of the steel rods ([10]; [11]; [9]). The equation 10 in this work can predict the % elongation of different steel rod at close accuracy if existing conditions are the same with those in this work with a standard error of just 0.37.

5. Conclusion

The research ''Further Study of Stress-Strain Deformation of Some Structural Reinforcement Steel Rods with Different Diameters from a Mini Mill in Nigeria using Theoretical and Regression Analysis'' has been undertaken and the following conclusions drawn from the study:

- Specimen 1 or the 10 mm diameter reinforcement steel rod exhibited a stress-strain deformation pattern that agrees with that of a ductile material. It had 9.36% elongation at failure and an ultimate tensile strength of 714MPa. The theoretical analysis shows that for a plain carbon steel with that high amount of ultimate tensile strength the elongation is adequate and relates to the % elongation at failure of the specimen.
- Three regression models where used viz: hyperbolic curve model, exponential function model and linear regression model to establish the nature of relationship between the diameter sizes of the steel rods, the % elongation at failure, and the ultimate tensile strength, and from the relationships develop prediction models
- The result showed that the relationship between the diameter sizes, the ultimate tensile strength and the % elongation of the reinforcement steel rods was more of a linear function.
- Prediction models were developed from the linear regression model as follows: $Y_1 = 9.41 - 0.009X \; and \; Y_2 = 897.315 - 25.48X$.
- $Y_{2=897.315-25.48X \; had \; a \; high \; degree \; of \; standard \; error \; of \; 60.18}$
- Coefficient of determination r2 shows that 21% of the variation in the % elongation of the steel rod may be predicted by change in the actual values of the reinforcement steel rod diameter. Other factors are responsible for the 79%.
- Coefficient of determination r2 shows that 1.49% of the variation in the ultimate tensile strength can be associated with a change in the diameter of the reinforcement steel bar.
- Standard error of regression for model equation Y1 is 60.18
- For the two model equations it has been observed that when individual prediction of the values of Y are made the confidence interval are much wider than the general confidence interval
- Standard error of the intercept (a) and the gradient (b) for model Y1 are 0.46 and 0.024.
- Model equation Y2 has a confidence interval of $897.315 \; \pm 1.277 \; for \; the \; intercept \; and - 25.48 \pm 0.0666 \; for \; the \; gradient$
- The rank correlation coefficient is -1 for the association of the diameter size of the reinforcement steel rod with respect to the % elongation of the reinforcement steel rod at the point of failure. The association is negative. The rank correlation coefficient for the association between variation in steel bar diameter and ultimate tensile strength is also in the negative, which means as one variable increases the other decreases. This assertion agrees with existing stress-strain equations.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors are on the same page concerning the publication of this article; there is therefore no conflict of interest.

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