International Journal of Frontiers in
Engineering and Technology Research
Journal homepage: https://frontiersrj.com/journals/ijfetr/
ISSN: 2783-0497 (Online)

(Research Article)

Mathematical modelling and simulation investigation of the dynamic behaviour of a compliant 2-R robot by using N-E method Via Matlab/Simulink

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International Journal of Frontiers in Engineering and Technology Research, 2022, 03(01), 010–019

Publication history: Received on 11 July 2022; revised on 30 August 2022; accepted on 01 September 2022

Article DOI: https://doi.org/10.53294/ijfetr.2022.3.1.0048

Abstract

This paper presents a mathematical modeling and simulation investigation on the effect of the equilibrium position on the stability and the energy provided by the robot by proposing four simulation cases. No closed solution for this critical study has been reported. An explicit elbow down model of a 2-R robot has been modelled by adding passive springs. The authors in this paper develop the dynamic of a compliant 2-R robot by using the extended Newton-Euler (N-E) method. The dynamic simulation is investigated by using Matlab/Simulink based on motion with jerk zero at the start-stop path, which guarantees less vibration to the robot's articulations. The simulation of trajectory is realized by SolidWorks to import the results to Matlab/Simulink for the dynamical simulation. The simulation results show that the energy-saving and good robot stability can be achieved whenever the equilibrium position is close from the beginning of motion with avoiding the unstable phase of the robot during working.

Keywords: Power saving; Stability; Equilibrium position; Work; N-E

1. Introduction

Simulation of robot systems becomes very important and very popular, especially with the lowering of computers. The possibility to perform real-time simulations becomes particularly crucial in the later stages of the design process. The final design can be verified before one embarks on the costly and time-consuming process of building a prototype. The need for accurate and computationally efficient manipulator dynamics has been extensively emphasized in recent years. The modelling and simulation of robot systems by using various program software's will facilitate the process of designing, constructing, and inspecting the robots in the real world. Simulation is important for robot programmers in allowing them to evaluate and predict the behaviour of a robot, and in addition to verify and optimize the path planning of the process. Moreover, this will save time and money and play an important role in the evaluation of manufacturing automation. Being able to simulate opens a wide range of options, helping to solve many problems creatively. One can investigate, design, visualize and test an object before making it a reality [1-5]. In the robot's industry, power-saving became an important factor in recent years [1, 6- 11]. Practically, when springs are added to the robot to support the entire robot system at the equilibrium position compared with a rigid robot. In this case, the role of using springs only to support the robot at the equilibrium position to assure a power-saving. Adding passive or active springs to reach this goal plays an important role. On the other hand, these springs may inflict fatigue during the motion to the robot due to the additional work that can provide to the joints. Thus, we need to choose a good position to support the entire robot system. The choice of a good position to support the entire robot system may offer to the robot a power-saving and good stability during motion. In previous studies [8-10, 12], the authors did not take in consideration the influence of the equilibrium position on power saving and stability of robots during motion. At the same time, these two factors are

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closely related to obtaining good performance of the robot's arms. Furthermore, obtaining good stability is very important for assuring the possibility of dynamic system stability control [13]. In robotics dynamic, springs may be used to support the weight of the robot itself or its weight and a supplementary charge or to manage energy. In this paper, the focus is on the weight support [8-10]. This research presents a continuation of the work done by the authors in Takanishi-Lab [8-9]. In this lab the authors studied only the effect of the equilibrium position on the total work of walking bipedal robot by using the software AMEsim. These robots are widely used in industry. Therefore, it is very important to study the stability and determine the zones of stable and unstable operation of these mechanisms. At present, it has not been possible to find scientific publications in which stability criteria are sufficiently clearly formulated in the study of these mechanisms. In this regard, the main objective of this paper is to find an optimal equilibrium position among several simulation cases to reach a power saving and good stability of a compliant 2- R mechanism by using N-E method. For this purpose, this paper is organized as follows: in section 2: modelling of a 2-R robot by adding passive springs, in section 3 is devoted to the discussion of obtained simulation results followed by the conclusion in section.

1.1. Mathematical Modelling of a Compliant 2-R Robot by Adding Passive Springs

In order to have an effective analysis on the important role of equilibrium position on power saving and stability of a 2-R robot during motion, we consider the 2-R compliant robot model shown in Figure 1 designed by using the software SolidWorks.

The 2-R robot mechanism shown below consists of four segments ($[AA']$, $[OO']$, $[O'O'']$ and $[BB']$). The center of mass is located at the center of each segment and "CMB" represents the body's center of mass. The two segments ($[AA']$ and $[BB']$) are considered in this study to remain parallel during motion and their weights are neglected in comparison with the weights of the two segments $[OO']$ and $[O'O'']$. The segment $[BB']$ represents the fixed element of robot and which is in contact with the ground. The gravitational force in our case is in inverse direction with the (y) axis. The springs ($\boldsymbol{S_1}$, $\boldsymbol{S_2}$) represent the springs used to support the entire robot at equilibrium position and ($\boldsymbol{F1}$, $\boldsymbol{F2}$) represent the forces generated in these springs respectively and can be calculated by the following Equation (1) [8-10]:

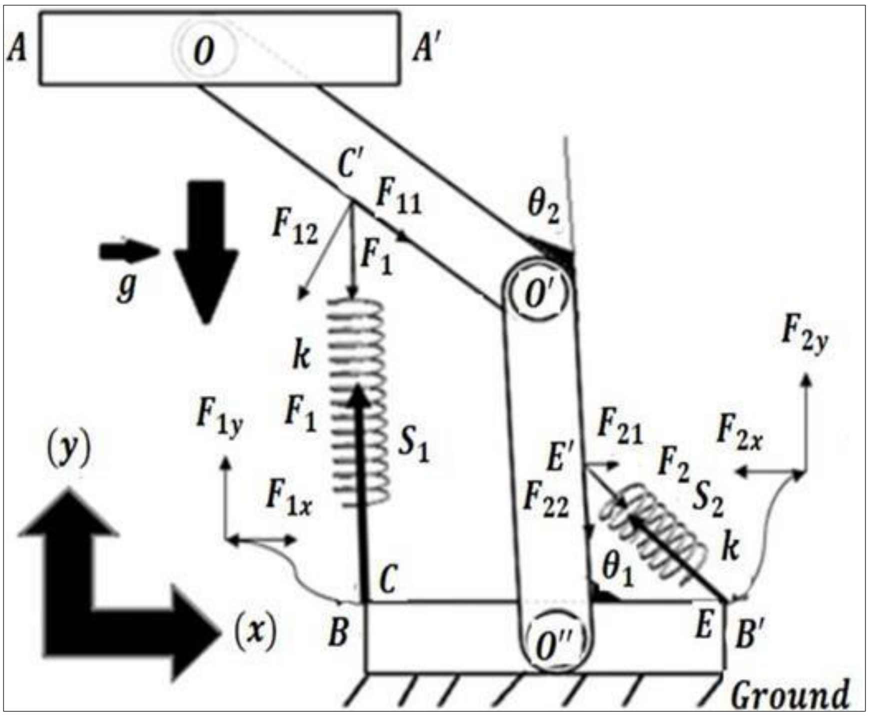

Figure 1 2-R robot modelled by adding passive springs using Solidworks

$$\boldsymbol{Fi} = \boldsymbol{ki}(\boldsymbol{xi} - \boldsymbol{ni}) - \boldsymbol{Di}\dot{\boldsymbol{x}}\boldsymbol{i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

The points (C, C', E and E') represent the spring attachments. For simplicity and easy of calculation, we put in our case the spring (CC') parallel to the segment $[O'O'']$ and the spring(EE') parallel to the segment $[OO']$.

1.1.1. Dynamic Modelling of a Compliant 2-R Robot

The Joint Torques

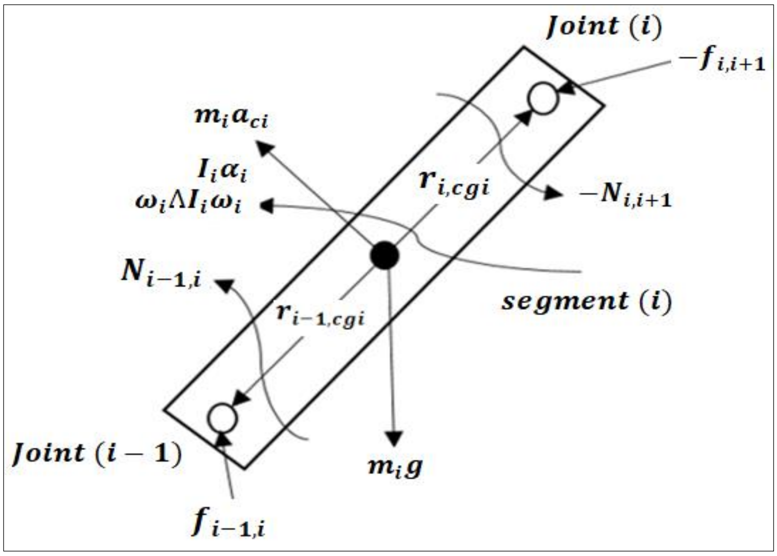

Figure 2 Representative body segment used in the inverse dynamics model

From segment positions and orientations, the net forces and torques acting on the joints are calculated recursively using the extended N-E method as shown in Figure 2. We call this method by the extended N-E because the spring forces are added to the joint torques. Thus, we need to develop the equations of joint torques according to spring forces. The computations are performed with Matlab/Simulink. By hand, we want to start at the joint ($\boldsymbol{O}$) and go down to the joint ($\boldsymbol{O''}$). N-E inverse dynamic is based on the following Equations (2) and (3) considering that the joints are massless:

$$\boldsymbol{fi-1},= \boldsymbol{fi,i+1} - \boldsymbol{mig} + \boldsymbol{miaci} + \boldsymbol{Fix} + \boldsymbol{Fiy} + \boldsymbol{F1i} + \boldsymbol{F2i} \text{ ....(2)}$$

$$Ni-1,i = Ni,i+1 - (cross(ri,cgi, fi,i+1)) + (cross(ri-1,cgi, fi-1,i)) + [I]i\alpha i + cross\ (\omega i, [I]i\omega i) \text{.......... (3)}$$

The cumulative work ($\boldsymbol{W}$) of each joint can be calculated as equation (4) and (5): [8-10]

$$W_i = \int_0^t |N_i \theta_i| dt \ / \ i = 1{,}2 \text{ .... (4)}$$

$$\boldsymbol{WT} = \sum \boldsymbol{Wi} \text{ ....(5)}$$

The Stability's Equation

The equation of motion of our model can be written as follows [13-15]:

$$\tau = (q)\dot{} + H(q)\dot{q} + L(q)\ where\ q(\theta 1, \theta 2) and\ \tau(\tau 1, \tau 2) \text{......................... (6)}$$

L can be written in the neighborhood of the equilibrium position considering the condition (7) is satisfied [15-17]:

$$\frac{\partial EP}{\partial Q} = \frac{0}{Q} = Q_0, \dot{Q}_0 = 0, Q_0 = \begin{bmatrix} \theta_{10} \\ \theta_{20} \end{bmatrix} \text{............................. (7)}$$

$$L = \frac{\partial^2 EP}{\partial Q^2} / Q = Q_0 \xrightarrow{yields} L = \begin{bmatrix} \frac{\partial^2 EP}{\partial \theta_{10}^2} & \frac{\partial^2 EP}{\partial \theta_{10}\theta_{20}} \\ \frac{\partial^2 EP}{\partial \theta_{20}\theta_{10}} & \frac{\partial^2 EP}{\partial \theta_{20}^2} \end{bmatrix} \text{....................................(8)}$$

Where;

$$PE = \frac{1}{2}(m_1 + m_2)gl_1s_1 + \frac{1}{2}m_2gl_2s_{12} + \frac{1}{2}k\Delta CC'^2 + \frac{1}{2}k\Delta EE'^2 \quad \text{..............................(9)}$$

$(\Delta CC')$, $(\Delta EE')$ represent the small displacements of the springs S_1, S_2respectively.

If all eigenvalues of $(L)$are positive, the stationary motion of the mechanical system corresponding to (6) is uniformly and asymptotically stable with respect to positional coordinates and their time derivatives. If matrix (L) has at least one negative eigenvalue, then the motion is unstable [15-17]. Among particular solutions of the system, at least one has negative Lyapunov characteristic number[15].The dynamic simulations are realized via Matlab/Simulink by using the extended Newton-Euler method. It is shown from the previous study that the software Solidworks can be used to calculate the kinematics and dynamics of mechanisms [1, 6-7, 18]. Thus, we need to simulate the trajectory of the compliant 2-R robot shown above by using Solidworks to import the results of displacements and velocities of springs to Matlab/Simulink to calculate the spring forces. In Matlab/Simulink PIDs are used to guarantee minimum errors between the nominal inputs which correspond to the results of SolidWorks and the realized outputs which correspond to the results of the dynamical simulation performed with Matlab/Simulink. The motion simulated by using inverse kinematics [1, 6-7, 18] of elbow down of a 2R robot corresponds to a motion in a straight line with a constant height (no vertical acceleration) with jerk zero at the start-stop path.

1.2. Simulation Investigation

This section represents the outcomes of work of the simulation of a compliant 2-R robot by using inverse kinematics. The 2-R robot moves between the two positions (1.5, 1) to (1.5, -1) with jerk zero at start-stop path during 1s. Generation of trajectories with a bounded value of jerk improves the tracking accuracy and will allow to reach a higher speed of task execution, with eventually a reduction in the excitation of the resonant frequencies and the vibrations caused by the planetary gear system [7, 19-22]. We can compare between the two postures of 2-R robot by using the equations of serial planar manipulators [18] in the dynamic modelling to choose the posture that has power saving [1, 6-7] and more stability.

In order to analyse the effect of equilibrium position on power saving and stability of our 2-R robot model during motion, four cases have been simulated with the same stiffness (k = 2500N/m) for the two springs (S1, S2) and we consider in aour case that at t = 0s the spring force equals to zero.

Case (1): No springs to support the entire robot system during motion.
Case (2): The two springs support the entire robot system at 0.3s.
Case (3): The two springs support the entire robot system at 0.5s.
Case (4): The two springs support the entire robot system at 0.7s.

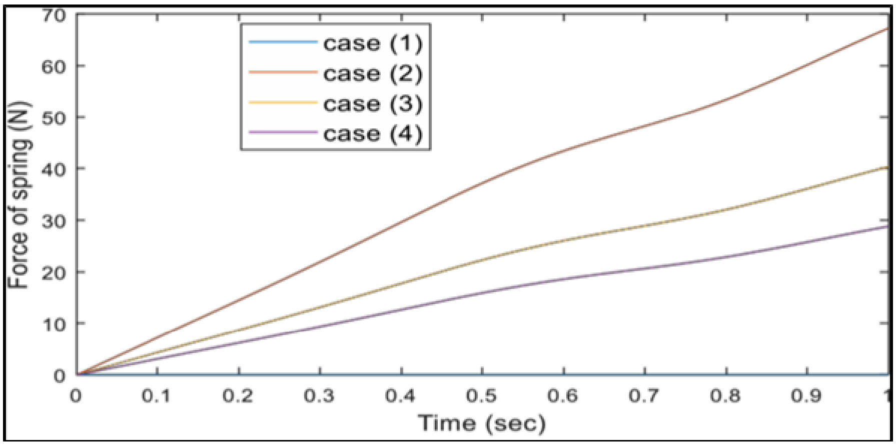

Figure 3 The variation of the force (F_1)

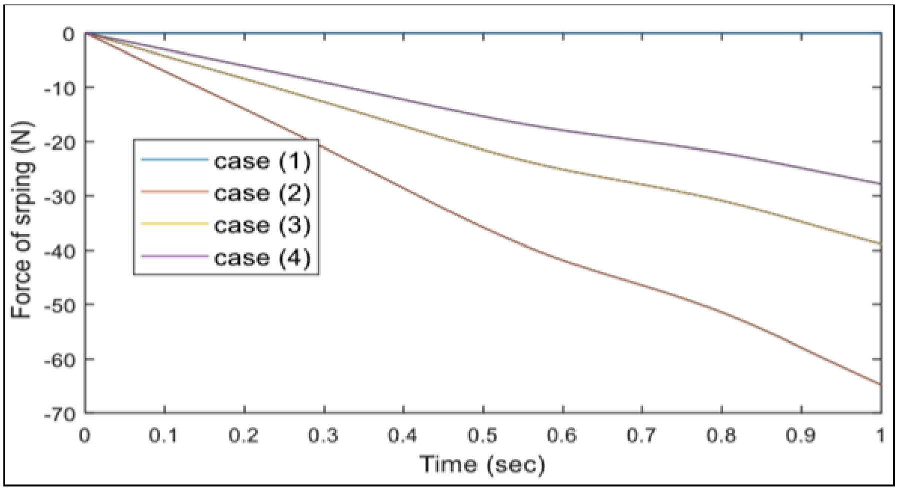

Figure 4 The variation of the force (F2)

Support the entire robot system at equilibrium position is realized by preload of the springs (S_1,S_2) (lengthening or compression). That means; in the equilibrium position with the nominal stiffness chosen, the robot is in static equilibrium. The parameters of simulation are shown in Table 1 and in our case, we consider that the segment's mass [AA'] is neglected in comparison with other segments.

Table 1 Parameters of simulation

Parameter	Value	Unit
$k1 = k2$	2500	N/m
$D1 = D2$	0.007	Ns/m
OO'	1	M
$O'O''$	1	M
OC'	1	M
$O'C'$	1	M
$m1 = m2$	2	Kg
$I1z = I2z$	0.01	Kg m2

The figures 3 and 4 show the variation of the spring forces during motion. The spring forces depend only on the joint angles. On Figures 3 and 4, we can see that the graphs are almost linear and the variation is due to intrinsic damping factor of springs (D_i). Another remark can be revealed from these figures is that the magnitude of spring forces (F_1, F_2) of the case (2) are higher than the other cases. This means that the springs of this case provides more forces than the other cases to support the entire system at this position.

Now we look at the obtained simulation results, and we start with the joint driving torques required to produce motion. Figures 5 and 6 show that the joint torque patterns of the two links (1) and (2) are the same for all cases. Another remark that can be added is that the two joint torques are in phase by 0°. The joint torque patterns of the link (1) show that the magnitude of the joint torque of the case (1) which correspond the dynamic simulation without springs is higher than the other cases and this is clear from the Figure (5). This means that using springs to support the entire robot system at the desired equilibrium position allows reducing the joint torque. Another fact that can be revealed from Figure (5) is that the joint torque can be reduced whenever the equilibrium position is close from the beginning of motion. It's clear from Figure (6)

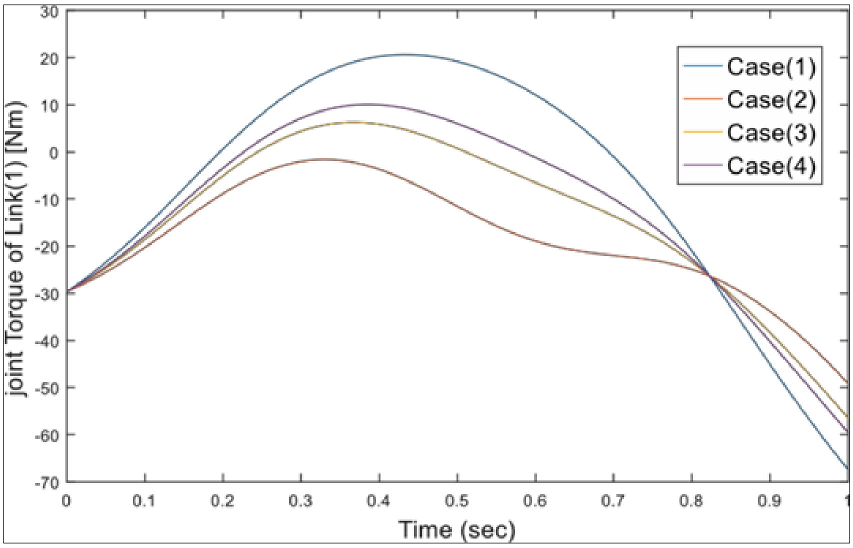

Figure 5 The joint torque of link (1)

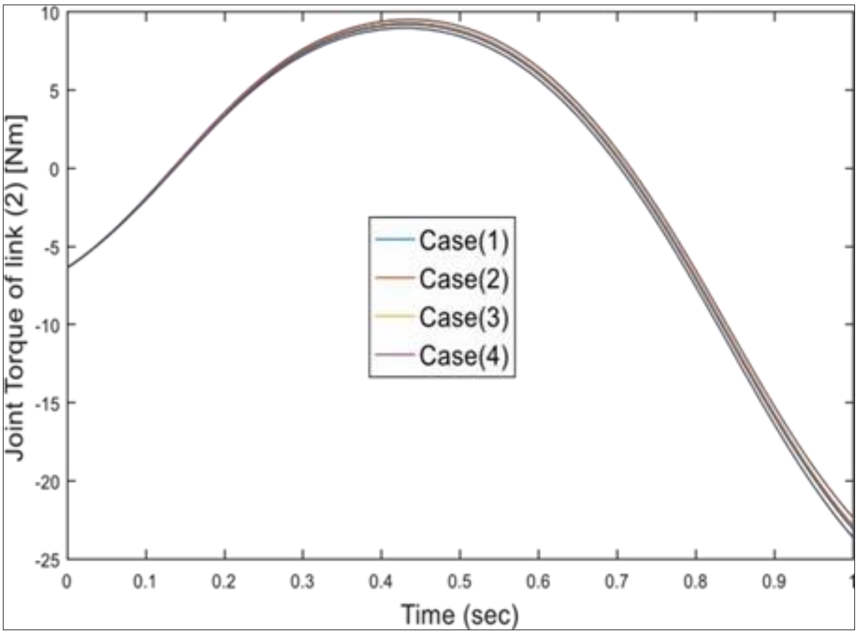

Figure 6 The joint torque of link (2)

That however the desired equilibrium position has no effect on the joint torque of the link (2), this is the same results show in [8-9] which correspond that the stiffness coefficient of springs has no effect on the joint torque of the knee (2). This means that on the link (2) the forces eliminate each other. We can conclude from the Figures (5) and (6) that the equilibrium position has only effect on the link (1).

The second point of focus is the total work done by the robot during motion. Figures (7) shows clearly the effect of equilibrium position on the mitigation of the total work of robot during motion. This mitigation is caused only by the work of the joint (1) because as shown from the joint torque analysis that the torque applied at the joint (2) is the same for all cases. It's shown from this figure (7) that the robot provides more energy when we don't use springs to support the entire robot at the equilibrium position. On the other hand, a good position plays an important role to choose which the position has the most power saving. For example, in our case it's clear from Figure (7) that the case (2) the robot provides less energy than the other cases and whenever we close to the end of motion the total work increases gradually, and this is also true for the two equilibrium positions (T=0.1s and T=0.9s). If we compare the two cases (1) and (2), we can notice that the work can be reduced by 68.22% at the end of the motion as shown the Table 2. This reduction guarantees to the robot very comfortable working during motion because the robot works with less energy.

Table 2 The total work (WT) at the end of motion

Case	WT(J) $at\ t = 1s$	**Reduction of work (case(1)-**
1	19.2	
2	6.1	68.22%
3	8	58.33%
4	12.2	36.45%

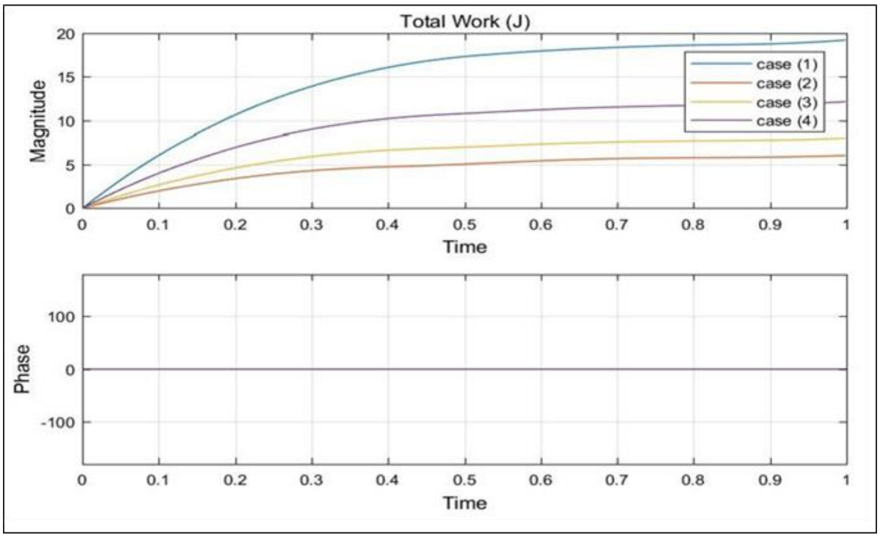

Figure 7 The total work done by the motors during motion

Finally, we analyse the stability of our 2-R robot model during motion for all cases. Figures (8) represents the robot's graph stability during 1s of the second eigen value. The first eigen value is not shown in this paper is positive real number. It is shown from Figure (8) that the robot is not stable for all cases in the interval [0-0.15s] (more precisely 0.1504s) even if it was stabilized before gyroscopic forces. This means that the robot takes a few times to reach the beginning of stability. From Figure (8), we can notice that the best stability can be chosen for the robot is the stability of the case (2). This is also true for the first eigen value.

Table 3 The stability at the end of motion

Case	$Stability\ (L)$ $at\ t = 1s$	**Increase of stability by ((case-case(1))/case(1))%**
1	8.32	
2	9.02	Appendix A. 8.41%
3	8.507	Appendix B. 0.96%
4	8.505	Appendix C. 2.24%

We can also notice from Figure (8) that the stability is like the work whenever the equilibrium position is close from the beginning of motion whenever a good stability can be reached. If we analyze only the second eigenvalue and if we compare between the two cases (1) and (2) we can remark that the stability increases by around 8.5% at the end of motion as shown the Table 3. Reducing the work by 68% and increasing the stability by 8.5% means that this position (case (2)) offers to the robot a good performance and very comfortable working during motion. These important results add an added value to the results shown in [810, 12] which correspond that the equilibrium position is when the hip joint (O) has the same coordinate in (x) with the ankle joint (O''). In our case, these two joints have the same coordinate at T=0.1567s, and the above results show that the equilibrium position at T=0.3s has more power-saving and best stability in comparison with the other cases. Another fact is revealed from this study and is not shown in this paper is that the equilibrium position at T=0.2s has the most power-saving and the best stability in comparison with the equilibrium position at T=0.3s. The use of springs as shown in our model to self-support the weight of the robot may be interesting especially when the robot would have to work more than 20% of time, the use of springs is recommended, based on the energy and stability criterions. Other configurations and paths pattern may also lead to different analysis and one must be careful in extrapolating the results presented here. It could be very interesting to study the effect of the equilibrium position on walking bipedal robots to show the effect of free leg on the stability of these robots. This opens the door to future works. It could be interesting also if a mechanism could be used for our model to engage/disengage springs in a way to manage energy while the robot is not in support phase to reduce more the energy.

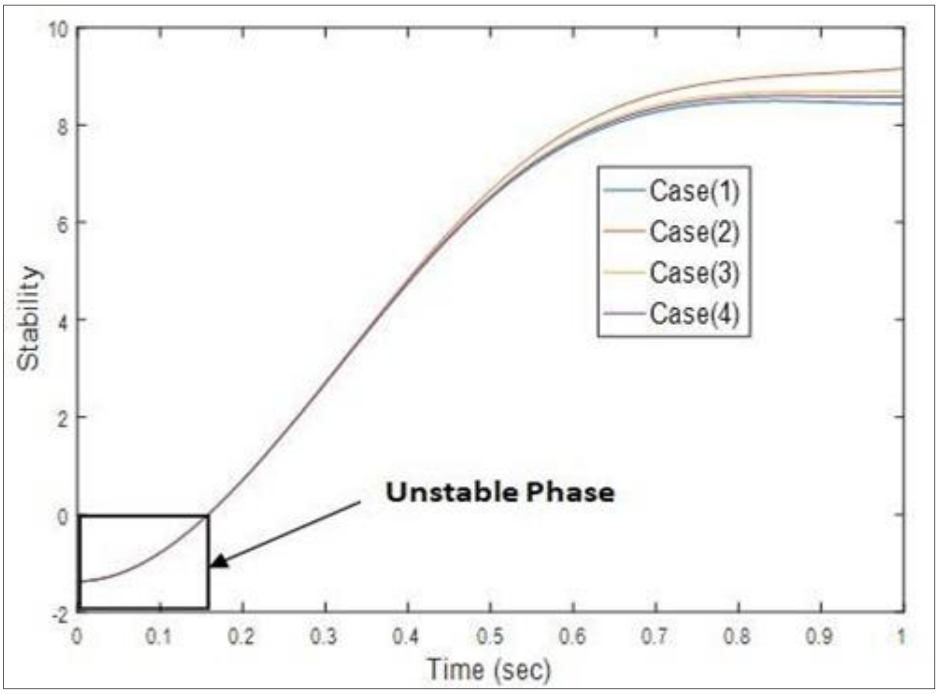

Figure 8 The robot's graph stability for all cases

1.3. Nomenclature

F_i: $The\ spring\ force.$
F_{ix}, F_{iy}: $The\ components\ of\ the\ force\ F_i\ applied\ at\ the\ joint\ (o'')/i = 1{,}2$
F_{1i}: $The\ components\ of\ the\ force\ F_1\ applied\ at\ the\ joint\ (o')/i = 1{,}2$
F_{2i}: $The\ components\ of\ the\ force\ F_2\ applied\ at\ the\ two\ joints\ (o'\ and\ o'')/i = 1{,}2$
x_i: $The\ lenght\ of\ the\ spring\ (i)$
n_i: $The\ rest\ lenght\ of\ the\ spring\ (i)$
D_i: $The\ interinsinc\ damping\ factor\ of\ the\ spring\ (i)$
$\dot{x}_i$: $The\ linear\ velocity\ of\ the\ spring\ (i)$
k_i: $The\ stiffness\ coefficient\ of\ the\ spring\ (i)$
$s_1 = \sin(\theta_1)$, $s_2 = \sin(\theta_2)$, $s_{12} = \sin(\theta_1 + \theta_2)$
$c_1 = \cos(\theta_1)$, $c_2 = \cos(\theta_2)$, $c_{12} = \cos(\theta_1 + \theta_2)$
l_i:$The\ lenght\ of\ the\ link\ (i)$
a_{ci}: $Linear\ acceleration\ vector\ of\ the\ center\ of\ mass\ of\ segment\ (i)$
$\dot{v}_i$:$Linear\ acceleration\ of\ the\ link\ (i)$
v_i: $Linear\ velocity\ of\ the\ link\ (i)$
$f_{i-1,}$: $Reaction\ force\ between\ segment\ (i-1) and\ (i)\ at\ joint\ (i-1)$
$f_{i,+1}$: $Reaction\ force\ vector\ between\ segments\ (i) and\ (i+1) at\ joint\ (i)$
$[I]_i$: $Inertia\ tensor\ [3x3]$
$[I]_{i,x,y,z}$: $Inertia\ about\ x, y, z$
m_i: $mass\ of\ segment\ (i)$
$N_{i-1,}$: $Reaction\ torque\ vector\ between\ segments\ (i-1)\ and\ (i)\ at\ joint\ (i)$
$N_{i,+1}$: $Reaction\ torque\ vector\ between\ segments\ (i) and\ (i+1)\ at\ joint\ (i)$

$r_{i,}$: $Vector\ from\ joint\ (i-1)\ to\ the\ center\ of\ mass\ of\ link\ (i)$
$r_{i-1,}$: $Vector\ from\ joint\ (i-1)\ to\ the\ center\ of\ mass\ of\ link\ (i)$
α_i:$Angular\ acceleration\ vector\ of\ segment\ (i)$
ω_i: $Angular\ velocity\ segment\ of\ link\ (i)$
$\dot{\theta}_i$: $Angular\ velocity\ of\ the\ joint\ angle\ (i)$
$\ddot{\theta}_i$: $Angular\ acceleration\ of\ the\ joint\ angle\ (i)$
g: $gravity$
$R_i^{i}{}_{-1}$: $Matrix\ transformation\ between\ links$
$P_i^{i}{}_{-1}$: $Position\ of\ joint\ (i)th\ respect\ to\ joint\ (i-1)$
t_i:$Torque\ applied\ at\ joint\ (i)$
τ_i: $Torque\ applied\ at\ joint\ (i)$
M: $Manipulator\ inertia\ matrix$
H: $Velocity\ coupling\ matrix$
L: $Gravitational\ force\ matrix$
PE: $Potential\ energy$
Q: $Generalized\ coordinate\ of\ the\ equilibrium\ position$
q: $generalized\ coordinates$
θ_{10}: $first\ equilibrium\ position\ with\ respect\ to\ the\ joint\ angle\ \theta_1$
θ_{20}: $second\ equilibrium\ position\ with\ respect\ to\ the\ joint\ angle\ \theta_2$

2. Conclusion

This paper addressed one of the scientific gaps of the dynamic of serial mechanisms via the dynamic analysis of a compliant 2-R robot by using N-E method. The verification of the obtained results by both softwares (Matlab/Simulink and AMEsim) allows us to qualitatively evaluate, underline the rightness of our mathematical modelling and to get the right conclusions. The effect of the equilibrium position on the power saving and stability of a compliant 2-R robot is analyzed in this research. Many simulation cases have been taken into consideration in this analysis. It is shown through this paper that the good equilibrium position for a power-saving and best stability is the position which is close to the beginning of motion with avoiding the time interval where the robot is in unstable phase. Obtaining these two factors power saving and good stability during motion, the two other factors are certainly obtained: comfortable working and good performance. A comparison of the obtained simulation results with other studies was not possible since these results have not been reported before.

Compliance with ethical standards

Acknowledgments

A part of this work has been done at Takanishi Lab-Tokyo (Japan). The author would like to thank Dr.Aiman Omer for his help and Pr benoit Levesque from Laval university (Canada) for the simulation performed with AMESIM . Also, thanks to the Laboratory of Mechanical Engineering, Materials and Structures-University Center of Tissemsilt for the laboratory and financial support.

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