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Correlation for predicting bubble point pressure for 22.3≤°API≥45 crude oils: A white-box machine learning approach

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Abstract

Bubble point pressure (BPP) is a key parameter for oil and gas reservoir identification, characterization, and management. An accurate correlation of this property with the evolving digital technology of machine learning, in the absence of experimental PVT analysis, serves as guidance in the development and recovery of reservoir fluids. In this study, a predictive BPP correlation was derived by intrinsically linearizing a nonlinear multiple regression, with the best coefficients (global minimum) extracted using White-box (Linear Regression, Ridge Regression, and Lasso Regression) Machine Learning models. The new correlation was developed, validated, and tested using 314 measured PVT data points from the Niger Delta Region. The data were subdivided into four classes: extra-light crude for API > 45, light crude for 31.1 < API ≤ 45, medium crude for 22.3 < API ≤ 31.1, and heavy crude for API ≤ 22.3. Statistical evaluation metrics such as root mean squared error, average absolute relative error, and average relative error were employed to compare the performance of the new correlation with the existing empirical ones. Results showed that the new BPP correlation developed by White-box Linear Regression outperformed the other White box (Ridge Regression and Lasso Regression) and other existing BPP empirical models. Taking advantage of emerging data-driven and machine learning as BPP predictive model is effective and efficient in reservoir fluids analysis.

Keywords: Bubble point pressure; Crude oil; White-box machine learning; Predictive model

1. Introduction

The physical and chemical properties of the oil within the Niger Delta have been reported to possess a high degree of variability all the way down to the reservoir system. The active source rock consists of the lower Agbada and also the upper Akata formations and corresponds to the middle Eocene period [1, 2]. Knowledge of the reservoir-fluid physical properties amongst other key factors is essential for production management, reservoir identification, and reservoir characterization. The characterization of reservoir fluid is essential in the estimation of reserves and prediction of reservoir performance. Inaccuracies in reservoir fluid characterization will result in errors in field development planning, uncertainty in the estimated prediction of oil and gas recovery, and thus in the overall asset value. It has been reported that one of the foremost methods of characterizing and predicting the behavior of reservoir fluids is the Pressure-Volume-Temperature (PVT) analysis [3-5].

Reservoir fluid Pressure-Volume-Temperature (PVT) empirical methods are quite expensive [3-6]; hence correlations have been developed to obtain these PVT properties. These correlations are highly region-dependent and rarely work well in different regions since they were developed with data from particular regions. Works of literature have shown that the predictive aptitude of the earlier PVT correlations developed for the Niger Delta crude oil system remains concomitant with huge errors hence, precise determination of these PVT properties is essential for

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the Niger Delta region. Among the pertinent reservoir properties of concern and great importance to the reservoir engineer is the bubble point pressure [5-16]. The bubble point pressure is a critical fluid property and is defined as the pressure at which the first bubble of gas appears at a specific temperature. It is required for the estimation of oil and gas reserves in material balance calculations and efficient reservoir management. The purpose of this paper is to demonstrate how machine learning can be used to develop a more accurate bubble point pressure correlation than is currently available.

A massive number of bubble point pressure empirical correlations by Standing in 1947 [7]; Lasater in 1958 [8]; Glaso in 1980 [9]; Al-Marhoun in 1988 [10]; Petrosky and Farshad in 1993 [11] were developed over the past six decades for diverse hydrocarbon systems. The majority of these models expressed bubble point pressure as a function of the reservoir temperature, solution gas-oil ratio, oil gravity, and gas-specific gravity. But the accuracy of these correlations is inadequate for the Niger Delta region owing to crude oil compositional differences from different regions of the world. The bubble point pressure empirical correlations for light oils (°API > 31) for the Niger Delta region were developed by Ikiensikimama and Ogboja, (2009) [12], Okoduwa and Ikiensikimama (2010) [13], Oloruntoba and Onyekonwu (2016) [14], and Jonathan and Joseph (2019) [15]; there is the need to develop correlations for extra-light, medium and heavy oils and improve the correlations for light oil for the Niger Delta region using Machine Learning model.

1.1. Machine Learning Models Description

There are two broad divisions of Machine Learning (ML) models namely the White-box ML models and Black-box ML models (Figure 1). According to Conor O'Sullivan (2020) [16], the white-box ML models also known as Interpretable models are relatively simple, can model linear relationships in data, and are easily understood by humans; their weights and bias terms can be extracted from them to give empirical formulas. Black-box ML models on the other hand can decipher complex relationships in data and can tell us how the different input features affect the result using feature importance plots, but they are however too complex for extracting equations [16]. The black-box models are also known as explainable models. In the issue of trust, the white-box model has been found to be more interpretable than the complex black-box model [16] and is therefore preferable. Also, the accuracy of every model is primarily dependent on the data utilized, and for data with few features or parameters such as the one utilized in this study, it has been observed that simpler white-box ML models give satisfactory results than the more complex black-box ML models. With these facts in check, the white box ML models provide a balanced trade-off between interpretability and accuracy (Conor O'Sullivan, 2020) [16] and were therefore utilized in this study.

(Source: https://towardsdatascience.com/interperable-vs-explainable-machine-learning-1fa525e12f48)

Figure 1 White-box Interpretable and Black-box Explainable models

Numerous researchers Alimadadi et al. (2011) [17]; Sanjari and Lay (2012) [18]; Kamari et al. (2013) [19]; Talebi et al. (2014) [20]; Aref et al. (2018) [21]; Sola Aremu (2019) [22] have used the black-box machine learning model to develop PVT properties. Aref et al (2018) [21] developed an Artificial Neural Network model to predict the bubble point pressure of crude oil. Their model was developed using 760 experimental data points from oil fields around the world. They presented accurate results in predicting bubble point pressure, but their work does not present any empirical correlation which is necessary for real-time practical usage. This lack of real-time availability is one of the weaknesses of using a black-box Machine Learning model. It is, therefore, appropriate to develop a correlation for bubble point pressure using a white-box Machine Learning model.

The main objective of the work, therefore, is to develop bubble point pressure correlation for extra-light, light, medium, and heavy crude oils for the Niger Delta using White-box Machine Learning models.

2. Material and methods

2.1. Data Description and Analysis

For this study, a total of 314 PVT data points from the Niger Delta region were collected to develop and validate the accuracy of the new correlation. The input data (Table 1) used consists of the solution gas-oil ratio (*Rsi*), reservoir temperature (T), API gravity (°API), oil-specific gravity (γ_0), and gas-specific gravity (γ_8). The data were partitioned or split into three (3) separate sets: the training set (60%), the validation set (10%), and the testing set (30%). Training sets are used to build models, whereas validation sets are used to guarantee that the developed models are produced most optimally, and testing sets are used to evaluate the final performance of the models.

Figure 2 Pair-Plot of data distribution

The PVT parameters were plotted together to give an overview of the data distribution. The pair plot (Figure 2) revealed a linear correlation between the solution gas-oil ratio and the bubble point pressure. Additionally, the plot indicated that the gas specific gravity was the least correlated feature to the solution gas-oil ratio in comparison to the other parameters; thus, combining them would not only eliminate the error of collinearity but would also increase the likelihood of a better correlation with the bubble point pressure.

Table 1 Range of data used for the study

True to inference from the observation of the distribution of the parameters, a ratio of the solution GOR to the gasspecific gravity gave a higher Pearson correlation with the bubble point pressure (0.67) than their individual correlations to the bubble point pressure (0.66 and -0.046). This is illustrated in the correlation plot below (Figure 3), this statistical inference is consistent with the Standing (1947) empirical method where he expressed the solution gasoil ratio and the gas specific gravity as ratio. Thus, data analysis enabled us to demonstrate statistically what was observed in laboratory measurements.

Figure 3 Pearson correlation of the PVT properties

2.2. Model Description

In this study, the White-box Machine Learning models (Linear regression, Ridge, and Lasso) were used to develop the bubble point pressure correlation for Nigeria crude oils. The process of model development by the White-box Machine Learning is as shown in Fig. 4.

2.3. Linear Regression

This is an example of supervised statistical learning. It is a linear model that assumes a linear relationship between a single dependent (output) variable (y), independent variables (x₁, x₂, x₃, etc.) used as input features and a bias term. Therefore, the aim of linear regression is to develop a model of continuous variable y as a function of one or more dependent variable(s), x, so that y can be predicted using the regression model when the only known is x (Jason, 2016) [23].

2.4. Ridge Regression

Ridge Regression (also known as Tikhonov regularization) is a regularized version of Linear Regression that includes a regularization parameter which helps keep the model's weights as small as possible. This included parameter controls the regularization of the model. For a regularization parameter value equal to zero, the ridge defaults to a simple Linear Regression model, for larger values of the regularization parameter, the weights end up very close to zero.

Figure 4 Flow Chart of the White-Box Machine Learning Process

2.5. Lasso Regression

Least Absolute Shrinkage and Selection Operator (Lasso) Regression is another regularized version of Linear Regression, and just like the Ridge Regression, it adds a regularization term to the cost function, but it uses a different normalization (ℓ 1 norm) of the weight vector instead of half the square of the ℓ 2 norm which is utilized in Ridge.

2.6. Model Development

In the development of the new correlation in this study, the bubble point pressure is expressed as a function of reservoir temperature, API gravity, and a ratio of solution GOR to gas specific gravity as shown in equation 1.

2 3 1 10 ⁰ *a a a g a s b API T R P* …………………………1

The nonlinear function (equation 1) was linearized by introducing logarithm (equation 2):

$$
\log P_b = a_0 + a_1 \log \left(\frac{R_s}{\gamma_g} \right) + a_2 \log API + a_3 \log T
$$

Equation 2 corresponds to the multiple regression equation (equation 3):

$$
y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + \varepsilon
$$

There is therefore the objective of finding optimum values of regression constants a_0 to a_3 which will cause the residual term ϵ to be negligible. In this study the optimal values of the unknown regression constants (a₀, a₁, a₂, and a₃) were obtained by utilizing the power of Linear Machine Learning algorithms to find the best constants (global minimum) that minimizes the residual term (ϵ) to zero.

2.7. Model Evaluation Analysis

In this study, graphical tool aids were used to explain the graphical error analysis and to check the new correlation's correctness and efficiency. The cross-plot approach was the primary technique used in this study. Also, three statistical measures, Average Relative Error (ARE), Average Absolute Relative Error (AARE), and Root Mean Squared Error (RMSE), were employed to validate and compare the accuracy of the new correlation with the existing ones.

3. Results and discussion

A significant factor in selecting a regression model was the requirement for a White-box machine learning model with extractable weights and bias terms (coefficients and constant) to establish an empirical correlation. 186 data points corresponding to 60% of the total data were used to train three linear Machine Learning models (white box) and 33 data points corresponding to 10% of the total data were used to validate the best performing model. With an average absolute error of 2.151 on the transformed validation set, the simple Linear Regression machine learning model outperformed the other two machine learning regression models (Ridge and Lasso). The unknown regression constants a₀ to a₃ were therefore extracted from the model. The new correlation developed is presented in equation (4).

The Figure 5 below is the cross-plot of the correlated and measured bubble point pressure. The line crosses the vast majority of the plotted points and produces the best-fitting lines.

Figure 5 Cross plot between the measured and the estimated bubble point pressure on the entire dataset using the new correlation developed

3.1. Comparing the new correlation with previous correlations:

The new bubble point pressure correlation developed in this study and the previously developed empirical bubble point pressures are as shown:

This study (2022):
$$
P_b = 10^{3.03928815} \left[\frac{R_s}{\gamma_g} \right]^{0.25715277} API^{0.24433212} \left(\frac{1}{T} \right)^{0.32761462}
$$

Standings (1947)[7]:

\n
$$
P_b = 18.2 \left[\left[\frac{R_s}{\gamma_g} \right]^{0.83} \times 10^{(0.00091T - 0.0125API)} - 1.4 \right]
$$

$$
P_b = \left(0.3841 - 1.2008\gamma_g + 7.522\gamma_g^2 \left(\frac{T^{\circ}R}{X}\right)\right)
$$

$$
X = \frac{R_s / 379.3}{\left(2.122 \times 10^{-11} \text{ J} \cdot \text{K} \cdot \text{
$$

 $(R_s/379.3)$ + $(350\gamma_o/M_o)$

 R_s /379.3) + (350 γ _o /M

 $^{+}$

 $(379.3) + (350\gamma)$

Lasater (1958)[8]:

$$
1.7669 + 1.7447 \log \left(\left(\frac{R_s}{\gamma_g} \right)^{0.816} \frac{T^{0.172}}{API^{0.989}} \right) - 0.30218 \left(\log \left(\left(\frac{R_s}{\gamma_g} \right)^{0.816} \frac{T^{0.172}}{API^{0.989}} \right) \right)^2
$$

= 10

Glaso (1980)[9]: *b P*

Al-Marbon (1988)[10]:
$$
P_b = 0.0053808 \Re_s^{0.715082} \frac{1}{\gamma_g^{1.87784}} \gamma_o^{3.1437} T^{1.32657}
$$

Petrosky-FarshaI (1993)[11]:
\n
$$
P_b = 112.727 \left(\frac{R_s^{0.5774}}{\gamma_g^{0.8439} \times 10^{0.0007916API^{1.541} - 0.00004561T^{1.3911}}} - 12.34 \right)
$$

Oloruntoba-Onyekonwu (2016)[14]:
$$
P_b = 10^{1.45274} R_s^{0.71363} \frac{1}{\gamma_g^{1.89527}} \frac{1}{API^{0.58612}} T^{0.30388}
$$

Jonathan-Joseph (2019)[15]: $P_b = 67.3506 R_s^{0.070147} \gamma_g^{1.066621} \gamma_o^{2.313833} T^{0.3682024}$

The trained and test data were divided into classes based on the oil API gravity, and the estimated bubble point pressure of the correlations were compared with the actual bubble point pressure based on these statistical measures: Average Relative Error (ARE), Average Absolute Relative Error (AARE), and Root Mean Squared Error (RMSE). To truly validate the performance of the new model, the results presented in this study represent estimated correlations carried out on the test data.

3.2. Extra-Light Crude-oil: API > 45

Table 2 Statistical Metrics of White-box ML and empirical models for extra-light crude-oil in order of ascending RMSE values

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Results (Table 2) showed that the new white-box machine learning correlation performed better than the existing ones on all statistical measures, with the correlation by Oloruntoba and Onyekonwu (2016) [14] coming up as the next highperforming correlation. A bar plot showing their ranking based on RMSE score is plotted below (Figure 6).

Figure 6 Bar-plot of estimated bubble point pressures of extra-light crude

3.3. Light Crude-oil: 31.1 < API ≤ 45

Table 3 Statistical Metrics of White-box ML and empirical models for light crude-oil in order of ascending RMSE values

The new White-box machine learning correlation performed better in terms of RMSE and ARE values (Table 3). While the correlation by Oloruntoba and Onyekonwu (2016) [14] achieved the best AARE score. A bar plot showing their ranking based on RMSE is plotted below (Figure 7).

Figure 7 Bar-plot of estimated bubble point pressures of light crude

3.4. Medium Crude-oil: 22.3 < API ≤ 31.1

Table 4 Statistical Metrics of White-box ML and empirical models for medium crude-oil in order of ascending RMSE values

From the result above (Table 4), the correlation developed from the linear regression and Ridge white-box model performed better than other empirical models based on the RMSE metric, while correlations by Glaso (1980) [9], Al-Marhoun (1988) [10], and the newly developed correlation (Lasso) performed better than others across the other statistical measures (ARE and AARE). A bar plot showing their ranking based on RMSE values is plotted below in Figure 8.

Figure 8 Bar-plot of estimated bubble point pressures of medium crude

3.5. Heavy Crude-oil: API ≤ 22.3

Because of the small number of data points in this class, the train and test data were merged to give a better representation of the class.

Table 5 Statistical Metrics of White-box ML and empirical models for heavy crude-oil in order of ascending RMSE values

From the result shown above (Table 5), the correlation from the white box algorithms performed better than the existing empirical models in estimating bubble point pressures of heavy crude. A bar plot showing their ranking based on RMSE score is plotted below (Figure 9).

Figure 9 Bar-plot of estimated bubble point pressures of heavy crude

Nomenclature

Appendix

Average Relative Error (MRE)

It represents the mean relative deviation of the estimated or predicted values from the experimental values. It is defined mathematically by the formula:

Average Absolute Relative Error (AARE)

It measures the mean value of the absolute relative deviation of the estimated value from the experimental value, and expressed mathematical as:

$$
AARE = \frac{1}{n} \sum_{i=1}^{i=n} \left| \frac{B_{\exp} - B_{est}}{B_{\exp}} \right|
$$

• Root Mean Squared Error (RMSE):

It is the square root of the average of the squared errors, given by the formula:

$$
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (B_{exp} - B_{est})^2}{n}}
$$
............A3

4. Conclusion

26 This study resulted in the development of a new correlation capable of accurately predicting bubble point pressure in the absence of experimental data. The correlation was developed by intrinsically linearizing a nonlinear equation and extracting the best constants from a White-box linear regression machine learning model fitted to the data. The new correlation was compared to other existing correlations, and the results indicated that the new model performed satisfactorily across the crude oil API gravity ranges. Therefore, in the absence of experimental measurements, the newly developed correlation is recommended for the prediction of bubble point pressure across the oil API gravity ranges, thus saving both the time and cost of the engineer and ultimately reducing the risk in the estimation of oil and gas reserve.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors declared that no conflict of interest exists.

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